Technical appendix (not for publication)

"Estimating DSGE models using seasonally adjusted and unadjusted data"

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A Data sources

I report the sources of data used in Table 3 in the main text and Table C.1 in this Appendix. The data set spans the period 1965Q1 to 2004Q4. Whenever the data set is provided in monthly frequencies, I simply take the average to transform it into quarterly frequencies. Both seasonally adjusted and unadjusted National Income and Product Accounts data are downloaded from the Bureau of Economic Analysis website. Nominal GDP, nominal consumption (defined as the sum of personal consumption expenditures on nondurables and services), and nominal investment (defined as the sum of gross private domestic investment and personal consumption expenditure on durables) are divided by the civilian noninstitutional population, downloaded from the Bureau of Labor Statistics (BLS hereafter) website, to convert the variables into per capita terms. To convert the variables into real terms, I divide them by the consumer price index for all urban consumers, available on the BLS website.[1] Working hours are measured by aggregate weekly hours in total private industries (available on the BLS website) divided by the population. Real wages are measured by average hourly earnings of production workers in total private industries (available on the BLS website), divided by the CPI. Inflation rates are measured by changes in the CPI. I use the effective federal funds rates (downloaded from the Federal Reserve Board website) to measure the nominal interest rates.

B Solving the seasonal DSGE model

The equilibrium of the baseline seasonal DSGE model in the main text is characterized by the equilibrium conditions below (B.1). To solve for the equilbrium, I log-linearize the equilibrium conditions (B.3) around the seasonal steady states (B.2). After carefully stacking the log-linearized equilibrium conditions so that they are consistent with the seasonal orderings, I can obtain the law of motion of the economy using a standard solution method of linear rational expectations models. A few words on notation: variables with a tilde denote detrended variables ($\widetilde{A}_t = A_t/X_t$) and variables with a hat denote log deviations of variables from their seasonal steady states ($\widehat{B}_t = \ln(B_t/B_q)$).

[1] I use the CPI for measuring price level since the seasonally unadjusted GDP deflator is not available.

B.1 Equilbrium conditions

$$\frac{\widetilde{K}_t}{H_t} = \frac{\alpha}{1-\alpha}\left(\frac{\widetilde{w}_t}{r_t^k}\right) \tag{B.1.1}$$

$$mc_t = \frac{1}{z_t \alpha^\alpha (1-\alpha)^{1-\alpha}} (\widetilde{w}_t)^{1-\alpha} (r_t^k)^\alpha \tag{B.1.2}$$

$$p_t^* = \frac{\theta_p}{\theta_p - 1}\left(\frac{P_t^n}{P_t^d}\right) \tag{B.1.3}$$

$$P_t^n = \widetilde{\lambda}_t mc_t \widetilde{Y}_t + \xi_p \beta \mathrm{E}_t \left(\frac{\pi_t^{\chi_p} \pi^{1-\chi_p}}{\pi_{t+1}}\right)^{-\theta_p} P_{t+1}^n \tag{B.1.4}$$

$$P_t^d = \widetilde{\lambda}_t \widetilde{Y}_t + \xi_p \beta \mathrm{E}_t \left(\frac{\pi_t^{\chi_p} \pi^{1-\chi_p}}{\pi_{t+1}}\right)^{1-\theta_p} P_{t+1}^d \tag{B.1.5}$$

$$1 = (1-\xi_p)(p_t^*)^{1-\theta_p} + \xi_p \left(\frac{\pi_{t-1}^{\chi_p} \pi^{1-\chi_p}}{\pi_t}\right)^{1-\theta_p} \tag{B.1.6}$$

$$\widetilde{Y}_t = (\widetilde{p}_t)^{\theta_p} z_t \widetilde{K}_t^\alpha H_t^{1-\alpha} \tag{B.1.7}$$

$$(\widetilde{p}_t)^{-\theta_p} = (1-\xi_p)(p_t^*)^{-\theta_p} + \xi_p \left(\frac{\pi_{t-1}^{\chi_p} \pi^{1-\chi_p}}{\pi_t}\right)^{-\theta_p} \tag{B.1.8}$$

$$\widetilde{\lambda}_t = \frac{\gamma \tau_t}{\gamma \widetilde{C}_t - b\widetilde{C}_{t-1}} - \beta b \mathrm{E}_t \left(\frac{\tau_{t+1}}{\gamma \widetilde{C}_{t+1} - b\widetilde{C}_t}\right) \tag{B.1.9}$$

$$1 = \beta \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}\left(\frac{R_t}{\gamma \pi_{t+1}}\right)\right] \tag{B.1.10}$$

$$\begin{aligned} \widetilde{\lambda}_t = & \widetilde{\psi}_t \mu_t \left[1 - S\left(\frac{\gamma \widetilde{I}_t}{\widetilde{I}_{t-1}}\right) - S'\left(\frac{\gamma \widetilde{I}_t}{\widetilde{I}_{t-1}}\right)\frac{\gamma \widetilde{I}_t}{\widetilde{I}_{t-1}}\right] \\ & + \beta \mathrm{E}_t \left[\frac{\widetilde{\psi}_{t+1}}{\gamma} \mu_{t+1} S'\left(\frac{\gamma \widetilde{I}_{t+1}}{\widetilde{I}_t}\right)\left(\frac{\gamma \widetilde{I}_{t+1}}{\widetilde{I}_t}\right)^2\right] \end{aligned} \tag{B.1.11}$$

$$\gamma \widetilde{\psi}_t = \beta \mathrm{E}_t [\widetilde{\lambda}_{t+1} r_{t+1}^k u_{t+1} + \widetilde{\psi}_{t+1}(1 - \delta(u_{t+1}))] \tag{B.1.12}$$

$$\widetilde{\lambda}_t r_t^k = \widetilde{\psi}_t \delta'(u_t) \tag{B.1.13}$$

$$\widetilde{K}_t = u_t \widetilde{K}_{t-1}^p \tag{B.1.14}$$

$$\gamma \widetilde{K}_t^p = (1 - \delta(u_t))\widetilde{K}_{t-1}^p + \mu_t \left(1 - S\left(\frac{\gamma \widetilde{I}_t}{\widetilde{I}_{t-1}}\right)\right)\widetilde{I}_t \tag{B.1.15}$$

$$f_t^1 = f_t^2 \tag{B.1.16}$$

$$f_t^1 = (\widetilde{w}_t^*)^{1-\theta_w} \widetilde{\lambda}_t H_t \widetilde{w}_t + \xi_w \beta \mathrm{E}_t \left(\frac{\pi_t^{\chi_w} \pi^{1-\chi_w} \widetilde{w}_t^*}{\widetilde{\pi}_{t+1}^w \widetilde{w}_{t+1}^*}\right)^{1-\theta_w} f_{t+1}^1 \tag{B.1.17}$$

$$f_t^2 = \frac{\theta_w}{\theta_w - 1} (\widetilde{w}_t^*)^{-\theta_w(1+\eta)} \varphi H_t^{1+\eta} + \xi_w \beta \mathrm{E}_t \left(\frac{\pi_t^w \pi^{1-\chi_w} \widetilde{w}_t^*}{\widetilde{\pi}_{t+1}^w \widetilde{w}_{t+1}^*}\right)^{-\theta_w(1+\eta)} f_{t+1}^2 \tag{B.1.18}$$

$$1 = (1-\xi_w)(\widetilde{w}_t^*)^{1-\theta_w} + \xi_w \left(\frac{\pi_{t-1}^{\chi_w} \pi^{1-\chi_w}}{\widetilde{\pi}_t^w} \right)^{1-\theta_w} \tag{B.1.19}$$

$$\widetilde{\pi}_t^w = \frac{\pi_t \widetilde{w}_t}{\widetilde{w}_{t-1}} \tag{B.1.20}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_R} \left\{ \left(\frac{\pi_t}{\pi_t^\star} \right)^{\phi_\pi} \left(\frac{\widetilde{Y}_t / \widetilde{Y}_{t-1}}{\widetilde{Y}_q / \widetilde{Y}_{q-1}} \right)^{\phi_Y} \right\}^{1-\rho_R} e^{\epsilon_{R,t}} \tag{B.1.21}$$

$$\widetilde{Y}_t = \widetilde{C}_t + \widetilde{I}_t + g_t \widetilde{Y}_t \tag{B.1.22}$$

$$\ln\left(\frac{z_t}{z_q} \right) = \rho_z \ln\left(\frac{z_{t-1}}{z_{q-1}} \right) + \epsilon_{z,t} \tag{B.1.23}$$

$$\ln\left(\frac{\tau_t}{\tau_q} \right) = \rho_\tau \ln\left(\frac{\tau_{t-1}}{\tau_{q-1}} \right) + \epsilon_{\tau,t} \tag{B.1.24}$$

$$\ln\left(\frac{\mu_t}{\mu_q} \right) = \rho_\mu \ln\left(\frac{\mu_{t-1}}{\mu_{q-1}} \right) + \epsilon_{\mu,t} \tag{B.1.25}$$

$$\ln\left(\frac{\pi_t^\star}{\pi_q} \right) = \rho_\pi \ln\left(\frac{\pi_{t-1}^\star}{\pi_{q-1}} \right) + \epsilon_{\pi,t} \tag{B.1.26}$$

$$\ln\left(\frac{g_t}{g} \right) = \rho_g \ln\left(\frac{g_{t-1}}{g} \right) + \epsilon_{g,t} \tag{B.1.27}$$

B.2 Seasonal steady states

$$\frac{\widetilde{K}_q}{H_q} = \frac{\alpha}{1-\alpha} \left(\frac{\widetilde{w}_q}{r_q^k} \right) \tag{B.2.1}$$

$$mc_q = \frac{1}{z_q \alpha^\alpha (1-\alpha)^{1-\alpha}} (\widetilde{w}_q)^{1-\alpha} (r_q^k)^\alpha \tag{B.2.2}$$

$$p_q^* = \frac{\theta_p}{\theta_p - 1} \left(\frac{P_q^n}{P_q^d} \right) \tag{B.2.3}$$

$$P_q^n = \widetilde{\lambda}_q mc_q \widetilde{Y}_q + \xi_p \beta \left(\frac{\pi_q^{\chi_p} \pi^{1-\chi_p}}{\pi_{q+1}} \right)^{-\theta_p} P_{q+1}^n \tag{B.2.4}$$

$$P_q^d = \widetilde{\lambda}_q \widetilde{Y}_q + \xi_p \beta \left(\frac{\pi_q^{\chi_p} \pi^{1-\chi_p}}{\pi_{q+1}} \right)^{1-\theta_p} P_{q+1}^d \tag{B.2.5}$$

$$1 = (1-\xi_p)(p_q^*)^{1-\theta_p} + \xi_p \left(\frac{\pi_{q-1}^{\chi_p} \pi^{1-\chi_p}}{\pi_q} \right)^{1-\theta_p} \tag{B.2.6}$$

$$\widetilde{Y}_q = (\widetilde{p}_q)^{\theta_p} z_q \widetilde{K}_q^\alpha H_q^{1-\alpha} \tag{B.2.7}$$

$$(\widetilde{p}_q)^{-\theta_p} = (1-\xi_p)(p_q^*)^{-\theta_p} + \xi_p \left(\frac{\pi_{q-1}^{\chi_p} \pi^{1-\chi_p}}{\pi_q} \right)^{-\theta_p} \tag{B.2.8}$$

$$\widetilde{\lambda}_q = \frac{\gamma \tau_q}{\gamma \widetilde{C}_q - b \widetilde{C}_{q-1}} - \beta b \left(\frac{\tau_{q+1}}{\gamma \widetilde{C}_{q+1} - b \widetilde{C}_q} \right) \tag{B.2.9}$$

$$1 = \beta\left[\frac{\lambda_{q+1}}{\lambda_q}\left(\frac{R}{\gamma\pi_{q+1}}\right)\right] \tag{B.2.10}$$

$$\widetilde{\lambda}_q = \widetilde{\psi}_q\mu_q\left[1 - S\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right) - S'\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right] + \beta\left[\frac{\widetilde{\psi}_{q+1}}{\gamma}\mu_{q+1}S'\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}\right)\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}\right)^2\right] \tag{B.2.11}$$

$$\gamma\widetilde{\psi}_q = \beta[\widetilde{\lambda}_{q+1}r^k_{q+1}u_{q+1} + \widetilde{\psi}_{q+1}(1 - \delta(u_{q+1}))] \tag{B.2.12}$$

$$\widetilde{\lambda}_q r^k_q = \widetilde{\psi}_q\delta'(u_q) \tag{B.2.13}$$

$$\widetilde{K}_q = u_q\widetilde{K}^p_{q-1} \tag{B.2.14}$$

$$\gamma\widetilde{K}^p_q = (1 - \delta(u_q))\widetilde{K}^p_{q-1} + \mu_q\left(1 - S\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\right)\widetilde{I}_q \tag{B.2.15}$$

$$f^1_q = f^2_q \tag{B.2.16}$$

$$f^1_q = (\widetilde{w}^*_q)^{1-\theta_w}\widetilde{\lambda}_q H_q\widetilde{w}_q + \xi_w\beta\left(\frac{\pi_q^{\chi_w}\pi^{1-\chi_w}\widetilde{w}^*_q}{\widetilde{\pi}^w_{q+1}\widetilde{w}^*_{q+1}}\right)^{1-\theta_w} f^1_{q+1} \tag{B.2.17}$$

$$f^2_q = \frac{\theta_w}{\theta_w - 1}(\widetilde{w}^*_q)^{-\theta_w(1+\eta)}\varphi H_q^{1+\eta} + \xi_w\beta\left(\frac{\pi^w_q\pi^{1-\chi_w}\widetilde{w}^*_q}{\widetilde{\pi}^w_{q+1}\widetilde{w}^*_{q+1}}\right)^{-\theta_w(1+\eta)} f^2_{q+1} \tag{B.2.18}$$

$$1 = (1 - \xi_w)(\widetilde{w}^*_q)^{1-\theta_w} + \xi_w\left(\frac{\pi_{q-1}^{\chi_w}\pi^{1-\chi_w}}{\widetilde{\pi}^w_q}\right)^{1-\theta_w} \tag{B.2.19}$$

$$\widetilde{\pi}^w_q = \frac{\pi_q\widetilde{w}_q}{\widetilde{w}_{q-1}} \tag{B.2.20}$$

$$\widetilde{Y}_q = \widetilde{C}_q + \widetilde{I}_q + g\widetilde{Y}_q \tag{B.2.21}$$

B.3 Log-linearized equilibrium conditions

$$\widehat{\widetilde{K}}_t - \widehat{H}_t = \widehat{\widetilde{w}}_t - \widehat{r}^k_t \tag{B.3.1}$$

$$\widehat{mc}_t = -\widehat{z}_t + (1 - \alpha)\widehat{\widetilde{w}}_t + \alpha\widehat{r}^k_t \tag{B.3.2}$$

$$\widehat{p}^*_t = \widehat{P}^n_t - \widehat{P}^d_t \tag{B.3.3}$$

$$P^n_q\widehat{P}^n_t = \widetilde{\lambda}_q mc_q\widetilde{Y}_q(\widehat{\widetilde{\lambda}}_t + \widehat{mc}_t + \widehat{\widetilde{Y}}_t) + \xi_p\beta\left(\frac{\pi_q^{\chi_p}\pi^{1-\chi_p}}{\pi_{q+1}}\right)^{-\theta_p} P^n_{q+1}[\theta_p(\mathrm{E}_t\widehat{\pi}_{t+1} - \chi_p\widehat{\pi}_t) + \mathrm{E}_t\widehat{P}^n_{t+1}] \tag{B.3.4}$$

$$P^d_q\widehat{P}^d_t = \widetilde{\lambda}_q\widetilde{Y}_q(\widehat{\widetilde{\lambda}}_t + \widehat{\widetilde{Y}}_t) + \xi_p\beta\left(\frac{\pi_q^{\chi_p}\pi^{1-\chi_p}}{\pi_{q+1}}\right)^{1-\theta_p} P^d_{q+1}[(\theta_p - 1)(\mathrm{E}_t\widehat{\pi}_{t+1} - \chi_p\widehat{\pi}_t) + \mathrm{E}_t\widehat{P}^d_{t+1}] \tag{B.3.5}$$

$$0 = (1-\xi_p)(p_q^*)^{1-\theta_p}\widehat{p}_t^* + \xi_p\left(\frac{\pi_{q-1}^{\chi_p}\pi^{1-\chi_p}}{\pi_q}\right)^{1-\theta_p}(\chi_p\widehat{\pi}_{t-1} - \widehat{\pi}_t) \tag{B.3.6}$$

$$\widehat{\widehat{Y}}_t = \theta_p\widehat{\widehat{p}}_t + \widehat{z}_t + \alpha\widehat{\widehat{K}}_t + (1-\alpha)\widehat{\widehat{H}}_t \tag{B.3.7}$$

$$(\widetilde{p}_q)^{-\theta_p}\widehat{\widehat{p}}_t = (1-\xi_p)(p_q^*)^{-\theta_p}\widehat{p}_t^* + \xi_p\left(\frac{\pi_{q-1}^{\chi_p}\pi^{1-\chi_p}}{\pi_q}\right)^{-\theta_p}(\chi_p\widehat{\pi}_{t-1} - \widehat{\pi}_t) \tag{B.3.8}$$

$$\begin{aligned}\widetilde{\lambda}_q\widehat{\widehat{\lambda}}_t = & \frac{\gamma b\tau_q}{(\gamma\widetilde{C}_q - b\widetilde{C}_{q-1})^2}\widetilde{C}_{q-1}\widehat{\widehat{C}}_{t-1} - \left[\frac{\gamma^2\tau_q}{(\gamma\widetilde{C}_q - b\widetilde{C}_{q-1})^2} + \frac{\beta b^2\tau_{q+1}}{(\gamma\widetilde{C}_{q+1} - b\widetilde{C}_q)^2}\right]\widetilde{C}_q\widehat{\widehat{C}}_t \\ & + \frac{\gamma\beta b\tau_{q+1}}{(\gamma\widetilde{C}_{q+1} - b\widetilde{C}_q)^2}\widetilde{C}_{q+1}\mathrm{E}_t\widehat{\widehat{C}}_{t+1} \\ & + \frac{\gamma\tau_q}{\gamma\widetilde{C}_q - b\widetilde{C}_{q-1}}\widehat{\tau}_t - \frac{\beta b\tau_{q+1}}{\gamma\widetilde{C}_{q+1} - b\widetilde{C}_q}\mathrm{E}_t\widehat{\tau}_{t+1}\end{aligned} \tag{B.3.9}$$

$$0 = \mathrm{E}_t\widehat{\widehat{\lambda}}_{t+1} + \widehat{R}_t - \widehat{\widehat{\lambda}}_t - \mathrm{E}_t\widehat{\pi}_{t+1} \tag{B.3.10}$$

$$\begin{aligned}\widetilde{\lambda}_q\widehat{\widehat{\lambda}}_t = & \widetilde{\psi}_q\mu_q\kappa\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\left(\frac{3\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}} - 2\gamma\right)\widehat{\widehat{I}}_{t-1} \\ & + \widetilde{\psi}_q\mu_q\left[1 - \frac{\kappa}{2}\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}} - \gamma\right)^2 - \kappa\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}} - \gamma\right)\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\right](\widehat{\widehat{\psi}}_t + \widehat{\mu}_t) \\ & - \left[\widetilde{\psi}_q\mu_q\kappa\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\left(\frac{3\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}} - 2\gamma\right) + \frac{\beta}{\gamma}\widetilde{\psi}_{q+1}\mu_{q+1}\kappa\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}\right)^2\left(\frac{3\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q} - 2\gamma\right)\right]\widehat{\widehat{I}}_t \\ & + \frac{\beta}{\gamma}\widetilde{\psi}_{q+1}\mu_{q+1}\kappa\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}\right)^2\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q} - \gamma\right)\mathrm{E}_t\widehat{\widehat{\psi}}_{t+1} \\ & + \frac{\beta}{\gamma}\widetilde{\psi}_{q+1}\widetilde{\mu}_{q+1}\kappa\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}\right)^2\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q} - \gamma\right)\mathrm{E}_t\widehat{\mu}_{t+1} \\ & + \frac{\beta}{\gamma}\widetilde{\psi}_{q+1}\mu_{q+1}\kappa\left(\frac{\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q}\right)^2\left(\frac{3\gamma\widetilde{I}_{q+1}}{\widetilde{I}_q} - 2\gamma\right)\mathrm{E}_t\widehat{\widehat{I}}_{t+1}\end{aligned} \tag{B.3.11}$$

$$\begin{aligned}\gamma\widetilde{\psi}_q\widehat{\widehat{\psi}}_t = & \beta\widetilde{\lambda}_{q+1}r_{q+1}^k u_{q+1}\mathrm{E}_t\widehat{\widehat{\lambda}}_{t+1} \\ & + \beta\widetilde{\lambda}_{q+1}r_{q+1}^k u_{q+1}\mathrm{E}_t\widehat{\widehat{r}}_{t+1}^k \\ & + \beta\widetilde{\psi}_{q+1}\left[1 - \delta_0 - \delta_1(u_{q+1}-1) - \frac{\delta_2}{2}(u_{q+1}-1)^2\right]\mathrm{E}_t\widehat{\widehat{\psi}}_{t+1}\end{aligned} \tag{B.3.12}$$

$$0 = -\widetilde{\lambda}_q r_q^k(\widehat{\widehat{\lambda}}_t + \widehat{r}_t^k) + \widetilde{\psi}_q[(\delta_1 + \delta_2(u_q-1))\widehat{\widehat{\psi}}_t + \delta_2 u_q\widehat{u}_t] \tag{B.3.13}$$

$$\widehat{\widehat{K}}_t = \widehat{u}_t + \widehat{\widehat{K}}_{t-1}^p \tag{B.3.14}$$

$$\begin{aligned}\gamma\widetilde{K}_q^p\widehat{\widehat{K}}_t^p = & \left[1 - \delta_0 - \delta_1(u_q-1) - \frac{\delta_2}{2}(u_q-1)^2\right]\widetilde{K}_{q-1}^p\widehat{\widehat{K}}_{t-1}^p + \kappa\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}} - \gamma\right)\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\mu_q\widetilde{I}_q\widehat{\widehat{I}}_{t-1} \\ & - (\delta_1 + \delta_2(u_q-1))u_q\widetilde{K}_{q-1}^p\widehat{u}_t + \left(1 - \frac{\kappa}{2}\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}} - \gamma\right)^2\right)\mu_q\widetilde{I}_q\widehat{\mu}_t\end{aligned}$$

$$+\kappa\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}-\gamma\right)\left(\frac{\gamma\widetilde{I}_q}{\widetilde{I}_{q-1}}\right)\mu_q\widetilde{I}_q\widehat{\widetilde{I}}_{t-1} \tag{B.3.15}$$

$$\widehat{f}_t^1 =\widehat{f}_t^2 \tag{B.3.16}$$

$$\begin{aligned} f_q^1\widehat{f}_t^1 =&\left[(\widetilde{w}_q^*)^{1-\theta_w}\widetilde{\lambda}_qH_q\widetilde{w}_q+\xi_w\beta\left(\frac{\pi_q^{\chi_w}\pi^{1-\chi_w}\widetilde{w}_q^*}{\widetilde{\pi}_{q+1}^w\widetilde{w}_{q+1}^*}\right)^{1-\theta_w}f_{q+1}^1\right](1-\theta_w)\widehat{\widetilde{w}}_t^*\\ &+(\widetilde{w}_q^*)^{1-\theta_w}\widetilde{\lambda}_qH_q\widetilde{w}_q(\widehat{\widetilde{\lambda}}_t+\widehat{H}_t+\widehat{\widetilde{w}}_t)\\ &+\xi_w\beta\left(\frac{\pi_q^{\chi_w}\pi^{1-\chi_w}\widetilde{w}_q^*}{\widetilde{\pi}_{q+1}^w\widetilde{w}_{q+1}^*}\right)^{1-\theta_w}f_{q+1}^1[(1-\theta_w)(\chi_w\widehat{\pi}_t-\mathrm{E}_t\widehat{\widetilde{\pi}}_{t+1}^w-\mathrm{E}_t\widehat{\widetilde{w}}_{t+1}^*)+\mathrm{E}_t\widehat{f}_{t+1}^1] \end{aligned} \tag{B.3.17}$$

$$\begin{aligned} f_q^2\widehat{f}_t^2 =&-\left[\frac{\theta_w}{\theta_w-1}(\widetilde{w}_q^*)^{-\theta_w(1+\eta)}\varphi H_q^{1+\eta}+\xi_w\beta\left(\frac{\pi_q^{\chi_w}\pi^{1-\chi_w}\widetilde{w}_q^*}{\widetilde{\pi}_{q+1}^w\widetilde{w}_{q+1}^*}\right)^{-\theta_w(1+\eta)}f_{q+1}^2\right]\theta_w(1+\eta)\widehat{\widetilde{w}}_t^*\\ &+\frac{\theta_w}{\theta_w-1}(\widetilde{w}_q^*)^{-\theta_w(1+\eta)}\varphi H_q^{1+\eta}(1+\eta)\widehat{H}_t\\ &+\xi_w\beta\left(\frac{\pi_q^{\chi_w}\pi^{1-\chi_w}\widetilde{w}_q^*}{\widetilde{\pi}_{q+1}^w\widetilde{w}_{q+1}^*}\right)^{-\theta_w(1+\eta)}f_{q+1}^2[-\theta_w(1+\eta)(\chi_w\widehat{\pi}_t-\mathrm{E}_t\widehat{\widetilde{\pi}}_{t+1}^w-\mathrm{E}_t\widehat{\widetilde{w}}_{t+1}^*)+\mathrm{E}_t\widehat{f}_{t+1}^2] \end{aligned} \tag{B.3.18}$$

$$0 =(1-\xi_w)(\widetilde{w}_q^*)^{1-\theta_w}\widehat{\widetilde{w}}_t^*+\xi_w\left(\frac{\pi_{q-1}^{\chi_w}\pi^{1-\chi_w}}{\widetilde{\pi}_q^w}\right)^{1-\theta_w}(\chi_w\widehat{\pi}_{t-1}-\widehat{\widetilde{\pi}}_t^w) \tag{B.3.19}$$

$$\widehat{\pi}_t^w =\widehat{\pi}_t+\widehat{\widetilde{w}}_t-\widehat{\widetilde{w}}_{t-1} \tag{B.3.20}$$

$$\widehat{R}_t =\rho_R\widehat{R}_{t-1}+(1-\rho_R)[\phi_\pi(\widehat{\pi}_t-\widehat{\pi}_t^*)+\phi_Y(\widehat{\widetilde{Y}}_t-\widehat{\widetilde{Y}}_{t-1})]+\epsilon_{R,t} \tag{B.3.21}$$

$$(1-g)\widetilde{Y}_q\widehat{\widetilde{Y}}_t =\widetilde{C}\widehat{\widetilde{C}}_t+\widetilde{I}\widehat{\widetilde{I}}_t+g\widetilde{Y}_q\widehat{g}_t \tag{B.3.22}$$

$$\widehat{z}_t =\rho_z\widehat{z}_{t-1}+\epsilon_{z,t} \tag{B.3.23}$$

$$\widehat{\tau}_t =\rho_\tau\widehat{\tau}_{t-1}+\epsilon_{\tau,t} \tag{B.3.24}$$

$$\widehat{\mu}_t =\rho_\mu\widehat{\mu}_{t-1}+\epsilon_{\mu,t} \tag{B.3.25}$$

$$\widehat{\pi}_t^\star =\rho_\pi\widehat{\pi}_{t-1}^\star+\epsilon_{\pi,t} \tag{B.3.26}$$

$$\widehat{g}_t =\rho_g\widehat{g}_{t-1}+\epsilon_{g,t} \tag{B.3.27}$$

C Additional tables and figures

Table C.1 compares the business cycle statistics in the data and in the baseline DSGE model (Section 3 in the main text). The model tends to underpredict the volatility of both investment and wage growth and overstate the volatility of hours growth. The model also tends to underpredict the correlation of consumption growth with respect to output growth and overpredict the correlation of both investment and wage growth with respect to output growth. Nevertheless, overall the model is successful in replicating salient features of the U.S. aggregate data.

Table C.2 reports posterior estimates of the main experiment using Tramo-Seats-filtered and DSGE-based-filtered data (Section 4 in the main text). The Tramo-Seats filter and the DSGE-based filter deliver similar biases compared to the X-12-Arima filter. (See, for example, the biases in some of the key structural parameters, α, η, and κ.) As in Section 6 in the main text, I consider the quadratic loss function by Ferroni (2011) that measures overall distortions. For the four estimation experiments considered,

1. Seasonally unadjusted data: $QL = 0.0039$
2. X-12-Arima-filtered data: $QL = 0.0669$
3. Tramo-Seats-filtered data: $QL = 0.0455$
4. DSGE-based-filtered data: $QL = 0.0418$

The DSGE-based-filtered data deliver considerable biases, although the magnitude is slightly smaller than that of the Tramo-Seats filter. I also note that all other results in Section 4 (impulse responses, business cycle statistics, and policy analysis) are robust to the choice of a seasonal adjustment filter. Figure C.1 plots the log-likelihood profiles for the model parameters given seasonally adjusted and unadjusted data.

Tables C.3, C.4, and C.5 report some model statistics of the alternative models considered in Section 5 in the main text. I briefly comment on each alternative model.

1. No investment adjustment cost model: The calibrated seasonal shifts in investment technology are almost constant across seasons (Table C.3).[2] Also, the model significantly underpredicts the volatilities of real wage growth, the inflation rate, and the interest rate (Table C.5).

2. No wage rigidity model: The model overpredicts seasonality and volatility in real wage growth (Tables C.4 and C.5).

3. Capital adjustment cost model: The model does a good job of replicating data moments, although it slightly underpredicts the volatilities of investment and wage growth (Table C.5).

[2]Some readers may think that the distortions in the estimated parameters in the baseline model are driven by the seasonal shifts in investment technology. To address this issue, I recomputed the probability limits for the baseline model but this time fixed the steady-state investment technology level constant across seasons. The result remained basically unchanged.

4. Labor adjustment cost model: The model significantly overpredicts seasonality and volatility in real wage growth (Tables C.4 and C.5). Also, it underpredicts the correlation of seasonally adjusted hours and wage growth with respect to output growth (Table C.5).

References

Ferroni, F., 2011. Trend agnostic one-step estimation of dsge models. B.E. Journal of Macroeconomics (Advances) 11, Article 25.

Table C.1: Business cycle statistics

	Percent standard deviation		Corr. with output growth	
Series	Data	Model	Data	Model
Output growth	0.95	0.96	–	–
Consumption growth	0.59	0.57	0.63	0.38
Investment growth	3.21	2.77	0.66	0.89
Hours growth	0.86	0.99	0.63	0.54
Wage growth	0.55	0.41	0.56	0.77
Inflation rate	0.75	0.73	-0.48	-0.22
Interest rate	0.76	0.69	-0.34	-0.18

Notes: Moments are calculated by applying the X-12-Arima filter to the simulated data from the seasonal model, where the parameters are fixed at their true values. All simulations are based on 100 replications of artificial time-series of length 200.

Table C.2: Posterior estimates

Parameter	Description	True	Unadjusted	T-S	DSGE-based
α	Capital share	0.3	0.29	0.58	0.53
			(0.0120)	(0.0325)	(0.0361)
b	Habit persistence	0.7	0.73	0.66	0.67
			(0.0134)	(0.0255)	(0.0270)
η	Inverse Frisch elasticity	2	2.01	0.96	1.06
			(0.1287)	(0.1443)	(0.1569)
κ	Investment adjustment cost	1	1.03	1.44	1.55
			(0.0760)	(0.1681)	(0.1936)
ξ_p	Calvo price	0.6	0.60	0.56	0.56
			(0.0024)	(0.0073)	(0.0075)
ξ_w	Calvo wage	0.6	0.59	0.56	0.52
			(0.0061)	(0.0187)	(0.0230)
χ_p	Price indexation	0.3	0.30	0.34	0.31
			(0.0080)	(0.0322)	(0.0303)
χ_w	Wage indexation	0.3	0.30	0.33	0.35
			(0.0112)	(0.0309)	(0.0309)
ρ_R	Taylor rule smoothing	0.7	0.71	0.71	0.71
			(0.0178)	(0.0254)	(0.0230)
ϕ_π	Taylor rule inflation	1.7	1.69	1.97	1.85
			(0.1398)	(0.2413)	(0.2012)
ϕ_Y	Taylor rule output	0.2	0.22	0.29	0.26
			(0.0499)	(0.0732)	(0.0664)
ρ_z	Neutral technology	0.95	0.95	0.95	0.95
			(0.0020)	(0.0054)	(0.0056)
ρ_τ	Preference	0.95	0.94	0.97	0.97
			(0.0091)	(0.0050)	(0.0068)
ρ_μ	Investment technology	0.95	0.96	0.88	0.90
			(0.0103)	(0.0207)	(0.0210)
ρ_π	Inflation target	0.95	0.96	0.95	0.95
			(0.0066)	(0.0099)	(0.0103)
ρ_g	Government spending	0.95	0.95	0.27	0.52
			(0.0109)	(0.0903)	(0.1152)

(Table continues on the next page.)

Table C.2: Posterior estimates (continued)

Parameter	Description	True	Unadjusted	T-S	DSGE-based
$100\sigma_z$	Neutral technology	0.9	0.90	0.83	0.84
			(0.0452)	(0.0416)	(0.0422)
$100\sigma_\tau$	Preference	1.7	1.72	1.47	1.54
			(0.1010)	(0.1232)	(0.1356)
$100\sigma_\mu$	Investment technology	1.4	1.68	1.11	1.18
			(0.2194)	(0.1144)	(0.1348)
$100\sigma_\pi$	Inflation target	0.1	0.09	0.11	0.10
			(0.0120)	(0.0142)	(0.0142)
$100\sigma_R$	Monetary policy	0.1	0.10	0.12	0.11
			(0.0054)	(0.0065)	(0.0064)
$100\sigma_g$	Government spending	1	0.90	1.30	1.46
			(0.0472)	(0.0954)	(0.1464)
$\tilde{z}_1$	Neutral technology Q1	0.97	0.97	–	–
			(0.0005)		
$\tilde{z}_2$	Neutral technology Q2	0.97	0.97	–	–
			(0.0003)		
$\tilde{z}_3$	Neutral technology Q3	0.97	0.97	–	–
			(0.0001)		
$\tilde{\tau}_1$	Preference Q1	0.77	0.74	–	–
			(0.0126)		
$\tilde{\tau}_2$	Preference Q2	0.95	0.94	–	–
			(0.0023)		
$\tilde{\tau}_3$	Preference Q3	0.92	0.92	–	–
			(0.0037)		
$\tilde{\mu}_1$	Investment technology Q1	0.81	0.80	–	–
			(0.0121)		
$\tilde{\mu}_2$	Investment technology Q2	0.98	0.97	–	–
			(0.0034)		
$\tilde{\mu}_3$	Investment technology Q3	0.91	0.91	–	–
			(0.0069)		

Notes: The table reports the MCMC estimates of posterior means. Standard deviations are reported in parentheses. The following reparameterizations are used: $\tilde{z}_q = z_q/z_4$, $\tilde{\tau}_q = \tau_q/\tau_4$, and $\tilde{\mu}_q = \mu_q/\mu_4$ for $q = 1, 2, 3$.

Table C.3: Parameters that vary across quarters: alternative models

Parameter	Description	Q1	Q2	Q3	Q4	Average
Panel A: No investment adjustment cost						
z	Neutral technology	0.99	1.00	0.99	1.02	1.00
τ	Preference	0.85	1.04	1.01	1.10	1.00
μ	Investment technology	1.00	1.00	1.00	1.00	1.00
Panel B: No wage rigidity						
z	Neutral technology	0.99	1.00	0.99	1.02	1.00
τ	Preference	0.84	1.04	1.01	1.10	1.00
μ	Investment technology	0.87	1.06	0.99	1.08	1.00
Panel C: Capital adjustment cost						
z	Neutral technology	0.99	1.00	0.99	1.02	1.00
τ	Preference	0.85	1.04	1.01	1.10	1.00
μ	Investment technology	0.94	1.00	1.01	1.04	1.00
Panel D: Labor adjustment cost						
z	Neutral technology	1.00	1.00	0.99	1.02	1.00
τ	Preference	0.84	1.04	1.02	1.10	1.00
μ	Investment technology	0.87	1.05	0.99	1.09	1.00

Table C.4: Seasonal patterns: alternative models

Series	Q1	Q2	Q3	Q4
Panel A: No investment adjustment cost				
Output*	-6.32	3.18	0.59	2.55
Consumption	-5.46	2.11	0.51	2.85
Investment*	-8.20	5.55	0.76	1.89
Hours*	-3.92	3.05	1.60	-0.73
Wage growth	-0.51	0.16	0.05	0.30
Inflation rate	0.39	0.02	-0.05	-0.35
Interest rate	0.00	0.00	0.00	0.00
Panel B: No wage rigidity				
Output*	-6.37	3.08	0.61	2.69
Consumption	-5.58	2.07	0.58	2.94
Investment*	-8.12	5.30	0.68	2.14
Hours*	-3.87	2.96	1.55	-0.64
Wage growth	-4.11	7.50	-1.13	-2.26
Inflation rate	-0.02	0.66	0.16	-0.80
Interest rate	0.00	0.00	0.00	0.00
Panel C: Capital adjustment cost				
Output*	-6.38	3.08	0.61	2.69
Consumption	-5.59	2.07	0.58	2.94
Investment*	-8.12	5.31	0.68	2.14
Hours*	-3.87	2.95	1.55	-0.64
Wage growth	-0.53	0.17	0.06	0.30
Inflation rate	0.40	0.01	-0.06	-0.35
Interest rate	0.00	0.00	0.00	0.00
Panel D: Labor adjustment cost				
Output*	-6.37	3.08	0.61	2.69
Consumption	-5.59	2.07	0.58	2.94
Investment*	-8.11	5.30	0.68	2.14
Hours*	-3.87	2.96	1.55	-0.64
Wage growth	-13.95	26.45	-9.59	-2.92
Inflation rate	-0.95	2.04	0.30	-1.39
Interest rate	0.00	0.00	0.00	0.00

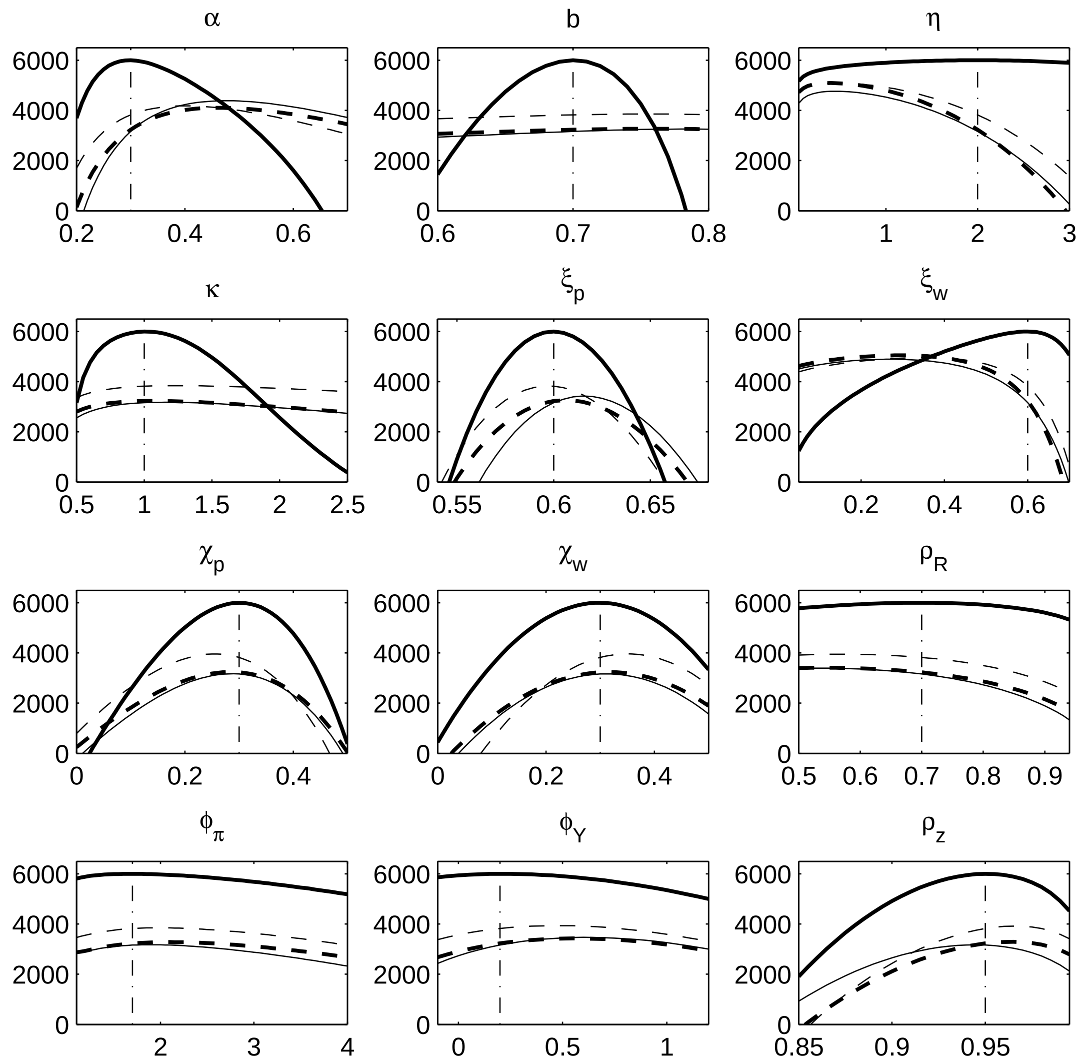
Notes: The table reports percent changes of variables from the previous quarter, taken from seasonal steady states in the model. Variables with * indicate those used as calibration targets.

Table C.5: Business cycle statistics: alternative models

Series	Percent standard deviation	Corr. with output growth
Panel A: No investment adjustment cost		
Output growth	0.91	–
Consumption growth	0.55	0.23
Investment growth	2.90	0.89
Hours growth	0.86	0.92
Wage growth	0.17	0.58
Inflation rate	0.13	-0.18
Interest rate	0.18	0.18
Panel B: No wage rigidity		
Output growth	0.98	–
Consumption growth	0.58	0.46
Investment growth	2.71	0.89
Hours growth	0.97	0.09
Wage growth	1.05	0.60
Inflation rate	0.73	-0.42
Interest rate	0.64	-0.32
Panel C: Capital adjustment cost		
Output growth	0.94	–
Consumption growth	0.58	0.73
Investment growth	2.13	0.90
Hours growth	0.74	0.42
Wage growth	0.43	0.79
Inflation rate	0.73	-0.39
Interest rate	0.67	-0.20
Panel D: Labor adjustment cost		
Output growth	0.96	–
Consumption growth	0.57	0.28
Investment growth	2.92	0.88
Hours growth	0.83	0.15
Wage growth	2.06	0.24
Inflation rate	0.72	-0.38
Interest rate	0.70	-0.26

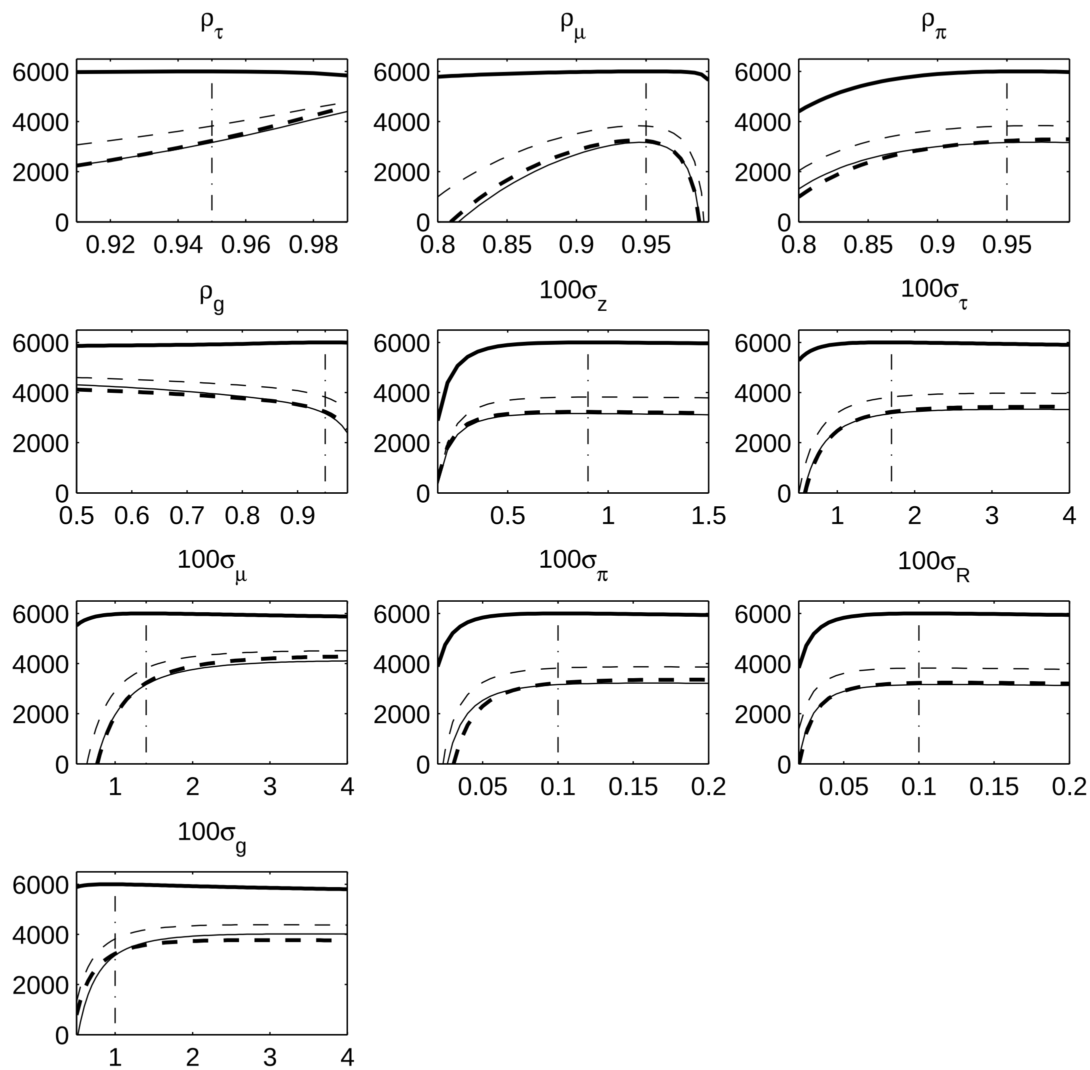
Notes: Moments are calculated by applying the X-12-Arima filter to the simulated data from the seasonal model, where the parameters are fixed at their true values. All simulations are based on 100 replications of artificial time-series of length 200.

Figure C.1: Likelihood profiles: seasonally adjusted vs. unadjusted data



(Figure continues on the next page.)

Figure C.1: Likelihood profiles: seasonally adjusted vs. unadjusted data (continued)



Notes: The figure plots likelihood profiles for seasonally unadjusted data (thick solid lines), X-12-Arima-filtered data (thick dashed lines), Tramo-Seats-filtered data (solid lines), and DSGE-based-filtered data (dashed lines). Vertical lines signify true values.