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Estimating DSGE models using seasonally adjusted and unadjusted data

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ABSTRACT

This paper evaluates the common practice of estimating dynamic stochastic general equilibrium (DSGE) models using seasonally adjusted data. The simulation experiment shows that the practice leads to sizable distortions in estimated parameters. This is because the effects of seasonality, which are magnified by the model's capital accumulation and labor market frictions, are not restricted to the so-called seasonal frequencies but instead are propagated across the entire frequency domain.

1. Introduction

Most aggregate time series display large seasonal fluctuations. As Barsky and Miron (1989) show, seasonal fluctuations account for a substantial fraction of total variations in quantity variables, such as GDP, investment, and hours worked. Nevertheless, the common practice among economists when estimating dynamic stochastic general equilibrium (DSGE) models is to simply ignore seasonality and use seasonally adjusted data. The practice implicitly assumes that seasonal adjustments can decompose data into seasonal and nonseasonal components, and values of interesting parameters can be recovered correctly. However, modern dynamic economic theory dictates that seasonality interacts with other endogenous variables in a complex and possibly nonlinear manner.[1] Hence seasonal adjustments based on arbitrary identifying restrictions would necessarily lead to distorted inference. An important question for macroeconomists is whether those distortions are quantitatively relevant.

In this paper, I develop a general equilibrium business cycle model that can account for broad features of U.S. seasonal and nonseasonal fluctuations. Building on recent contributions (e.g., Christiano et al., 2005; Smets and Wouters, 2007; Justiniano et al., 2010), the model incorporates a host of real and nominal frictions and various types of shocks. The model is also subject to seasonal variations in technology and preference. Endogenous responses by agents to those seasonal variations allow the model to reproduce the seasonality observed in the U.S. aggregate data. I then simulate artificial data from the parameterized model in order to analyze the effects of estimating DSGE models using seasonally adjusted data.

A hypothetical econometrician uses the seasonally adjusted data to estimate an aseasonal counterpart of the baseline model using Bayesian methods. I find that the estimated parameters differ substantially from their true values. In contrast, when estimated with seasonally unadjusted data, most parameters are very precisely estimated. The result is crucial, because it suggests that the conventional practice of estimating DSGE models may lead to severely biased inference and that policy experiments based on the estimated parameters could be misleading.

Given the significance of the finding, I devote considerable effort to studying the reasons for this distortion. Importantly, the distortions cannot be mitigated by constructing alternative seasonal adjustment filters, as they still arise in large sample environments with "ideal" filters. Using frequency domain tools, I show that the effects of seasonality are not confined to the so-called seasonal frequencies but instead are propagated across other nonseasonal frequencies. In particular, the effects are noticeable at higher frequencies and act in ways that raise spectral power in those regions. The intuition is relatively straightforward: Since seasonality induces agents to reallocate their resources across

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[1] For an elegant exposition of this issue, see Ghysels (1988).

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seasons within a year, the effects of seasonality are noticeable at higher frequencies. Moreover, because of seasonality, agents have different responses across seasons to the same shocks, and this additional source of volatility raises spectral power. I show that two key frictions in the model — the investment adjustment cost and the nominal wage rigidity — magnify the nonlinear interactions of seasonality and endogenous variables and make the propagation of the seasonal components quantitatively relevant. As a result, standard seasonal adjustment procedures that try to dampen spectral power only near seasonal frequencies are not effective, and the estimated parameters have to adjust in order to compensate for the discrepancy of spectra between seasonal and aseasonal versions of the model. I also provide some evidence suggesting that frictions that generate large distortions are not limited to those I assumed in the baseline model but include other general classes of capital accumulation and labor market frictions as well.

The present paper builds on several important contributions from the previous literature. Sims (1993) and Hansen and Sargent (1993) forcefully defend the common practice of estimating DSGE models using seasonally adjusted data. Their argument is based on two observations. First, directly modeling seasonality may lead to large distortions if the mechanism generating seasonality is misspecified. Second, in most examples they consider, using seasonally adjusted data leads to fairly accurate estimates. My contribution with respect to their papers is to show that, in a state-of-the-art DSGE model that is parameterized to match certain features of U.S. business cycle fluctuations, the second observation does not hold. I also deal with concerns about model misspecifications in more detail later in the paper. This paper is also related to Christiano and Todd (2002). There are two main departures from their study. First, they focus on the effects of seasonal adjustment on business cycle statistics. I consider the effects on a likelihood-based inference. Since a likelihood function contains all information from cross-equation restrictions imposed by dynamic economic theory, implications of seasonality may be quite different from those based on arbitrary sets of moments. Moreover, since it has now become a widely accepted approach to estimate DSGE model parameters using formal econometric methods, I believe this is a relevant application for many researchers. Second, they use a standard real business cycle model to answer their question at hand. My model introduces additional frictions and propagation structures (e.g., habit persistence, capital utilization, nominal rigidities, etc.) into their model. As I will show, some of the new added features in my model are the key driving force of my results.

The rest of the paper is organized as follows. Section 2 constructs a DSGE model with seasonality. Section 3 sets up the main experiment. Section 4 reports the results and shows that seasonal adjustments lead to sizable distortions in parameter estimates. Section 5 identifies reasons for the distortions. Section 6 proposes a practical procedure that helps researchers decide whether or not to include seasonality in their models when potential model misspecifications are of concern. Finally, Section 7 concludes.

2. The seasonal DSGE model

The baseline seasonal model builds on a medium-scale DSGE model with a number of real and nominal frictions, along the lines of Christiano et al. (2005), Smets and Wouters (2007), and Justiniano et al. (2010). Following the previous literature on the subject (e.g., Chatterjee and Ravikumar, 1992; Braun and Evans, 1995; Liu, 2000), seasonality originates from deterministic shifts in technology and preference. Variations in technology could represent, for example, seasonal fluctuations in weather. Variations in preferences could represent expenditures due to several kinds of social events, such as Christmas. Presumably modeling seasonality in such a way that it originates from deeper structures of the economy would strengthen the case for using seasonally unadjusted data. However, the question I would like to ask in this paper is whether even a seemingly innocuous, simple mechanism for seasonality would generate large distortions through the *endogenous* responses to seasonality by optimizing agents.

The economy is composed of the final-goods sector, intermediate-goods sector, household sector, employment sector, and a government. I will begin by describing the production side of the economy.

2.1. The final-goods sector

In each period t, the final goods, Y_t, are produced by a perfectly competitive representative firm that combines a continuum of intermediate goods, indexed by $j \in [0, 1]$, with technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\theta_p - 1}{\theta_p}} dj \right]^{\frac{\theta_p}{\theta_p - 1}}.$$

Here, $Y_{j,t}$ denotes the time t input of intermediate good j and θ_p controls the price elasticity of demand for each intermediate good. The demand function for good j is

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_p} Y_t,$$

where P_t and $P_{j,t}$ denote the price of the final good and intermediate good j, respectively. Finally, P_t is related to $P_{j,t}$ via the relationship

$$P_t = \left[\int_0^1 P_{j,t}^{1-\theta_p} dj \right]^{\frac{1}{1-\theta_p}}.$$

2.2. The intermediate-goods sector

The intermediate-goods sector is monopolistically competitive. In period t, each firm j buys $K_{j,t}$ units of capital service from the household sector and $H_{j,t}$ units of aggregate labor input from the employment sector to produce intermediate good j using technology

$$Y_{j,t} = z_t K_{j,t}^{\alpha} (X_t H_{j,t})^{1-\alpha},$$

where z_t is the neutral technology shock at time t. z_t follows the law of motion

$$\ln \left(\frac{z_t}{z_q} \right) = \rho_z \ln \left(\frac{z_{t-1}}{z_{q-1}} \right) + \epsilon_{z,t}, \quad \epsilon_{z,t} \sim N(0, \sigma_z^2),$$

where z_q is the steady-state level of z_t in season q. α is the capital share in the production function and X_t is a deterministic technological process that grows at rate γ.

In period t, the firm can reoptimize its intermediate-goods price with probability $(1 - \xi_p)$. Firms that cannot reoptimize index their price according to the following: $P_{j,t} = \pi_{t-1}^{\chi_p} \pi^{1-\chi_p} P_{j,t-1}$, where π_{t-1} is the inflation rate in period $t - 1$, π is the steady-state inflation rate (which is different from the steady-state level of the inflation rate in season q, π_q), and $\chi_p \in [0, 1]$ is a parameter that controls the degree of indexation to past inflation.

2.3. The household sector

There is a continuum of households, indexed by $i \in [0, 1]$. In each period, household i chooses consumption C_t, investment

I_t, bond purchases B_t, and nominal wage $W_{i,t}$ to maximize utility given by the following:

$$E_t \sum_{s=0}^{\infty} \beta^s \left[\tau_{t+s} \ln(C_{t+s} - bC_{t+s-1}) - \varphi \frac{H_{i,t+s}^{1+\eta}}{1+\eta} \right],$$

where β is a discount factor, b represents consumption habit, η controls (the inverse of) the Frisch labor supply elasticity, and $H_{i,t}$ is the number of hours worked by i. φ is a scale factor that determines hours worked in the steady state. I normalize $\varphi = 1$. τ_t is the preference shock that follows the process:

$$\ln\left(\frac{\tau_t}{\tau_q}\right) = \rho_\tau \ln\left(\frac{\tau_{t-1}}{\tau_{q-1}}\right) + \epsilon_{\tau,t}, \quad \epsilon_{\tau,t} \sim N(0, \sigma_\tau^2),$$

where τ_q is the steady-state level of τ_t in season q.

The household's budget constraint is

$$P_tC_t + P_tI_t + B_t \leq W_{i,t}H_{i,t} + R_t^k u_t K_{t-1}^p + R_{t-1}B_{t-1} + D_t + A_{i,t} + T_t$$

where R_t^k is the rental rate of capital, u_t is the utilization rate of capital, K_{t-1}^p is the stock of physical capital, R_{t-1} is the gross nominal interest rate from period $t-1$ to t, D_t is the combined profit of all the intermediate-goods firms distributed equally to each household, and T_t are lump-sum transfers from the government. I assume that households buy securities, whose payoffs are contingent on whether it can reoptimize its wage.[2] $A_{i,t}$ denotes the net cash inflow from participating in state-contingent security markets at time t.

Capital utilization transforms physical capital into capital services according to

$$K_t = u_t K_{t-1}^p.$$

The physical capital stock evolves according to the following law of motion:

$$K_t^p = (1 - \delta(u_t))K_{t-1}^p + \mu_t \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t.$$

Following Greenwood et al. (1988), I assume that increasing the intensity of capital utilization speeds up the rate of depreciation $\delta(u_t)$. As in Schmitt-Grohe and Uribe (2008), I adopt a quadratic formulation for the function δ:

$$\delta(u_t) = \delta_0 + \delta_1(u_t - 1) + \frac{\delta_2}{2}(u_t - 1)^2,$$

with $\delta_0, \delta_1, \delta_2 > 0$. The function S captures the notion of adjustment costs in investment, as proposed in Christiano et al. (2005). I adopt the following specification for S:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2,$$

with $\kappa > 0$. Finally, μ_t is the investment technology shock that follows the process:

$$\ln\left(\frac{\mu_t}{\mu_q}\right) = \rho_\mu \ln\left(\frac{\mu_{t-1}}{\mu_{q-1}}\right) + \epsilon_{\mu,t}, \quad \epsilon_{\mu,t} \sim N(0, \sigma_\mu^2),$$

where μ_q is the steady-state level of μ_t in season q.

2.4. The employment sector and wage setting

In each period t, a perfectly competitive representative employment agency hires labor from a continuum of households, indexed by $i \in [0, 1]$, to produce an aggregate labor service, H_t, using technology

$$H_t = \left[\int_0^1 H_{i,t}^{\frac{\theta_w - 1}{\theta_w}} di\right]^{\frac{\theta_w}{\theta_w - 1}},$$

where $H_{i,t}$ denotes the time t input of labor service from household i and θ_w controls the price elasticity of demand for each household's labor service. The agency sells the aggregated labor input to the intermediate firms for a nominal price of W_t per unit. The demand function for the labor service of household i is

$$H_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} H_t,$$

where $W_{i,t}$ denotes the nominal wage rate of the labor service of household i. W_t is related to $W_{i,t}$ via the relationship

$$W_t = \left[\int_0^1 W_{i,t}^{1-\theta_w} di\right]^{\frac{1}{1-\theta_w}}.$$

In each period t, a household faces a probability $(1 - \xi_w)$ of being able to reoptimize its nominal wage. Households that cannot reoptimize index their wage according to the following: $W_{i,t} = \gamma \pi_{t-1}^{\chi_w} \pi^{1-\chi_w} W_{i,t-1}$, where $\chi_w \in [0, 1]$ is a parameter that controls the degree of indexation to past inflation.

2.5. The government and resource constraint

The central bank follows a Taylor-type reaction function:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left\{ \left(\frac{\pi_t}{\pi_t^\star}\right)^{\phi_\pi} \left(\frac{Y_t/Y_{t-1}}{Y_q/Y_{q-1}}\right)^{\phi_Y} \right\}^{1-\rho_R} e^{\epsilon_{R,t}},$$

$$\epsilon_{R,t} \sim N(0, \sigma_R^2)$$

where R is the steady-state level of the nominal interest rate, ρ_R is the persistence of the rule, and ϕ_π and ϕ_Y are the size of the policy response to the deviation of inflation and output growth from their targets, respectively. Y_t/Y_{t-1} is the growth rate of output in period t and Y_q/Y_{q-1} is the steady-state growth rate of output in season q. $\epsilon_{R,t}$ is an exogenous shock to the interest rate rule. $\pi_t^\star$ is the central bank's inflation target, which evolves according to

$$\ln\left(\frac{\pi_t^\star}{\pi_q}\right) = \rho_\pi \ln\left(\frac{\pi_{t-1}^\star}{\pi_{q-1}}\right) + \epsilon_{\pi,t}, \quad \epsilon_{\pi,t} \sim N(0, \sigma_\pi^2),$$

where π_q is the steady-state inflation rate in season q.

The aggregate resource constraint is $C_t + I_t + G_t = Y_t$. G_t is the amount of government spending, which is determined as a time-varying fraction of output

$$G_t = g_t Y_t,$$

and g_t follows the process

$$\ln\left(\frac{g_t}{g}\right) = \rho_g \ln\left(\frac{g_{t-1}}{g}\right) + \epsilon_{g,t}, \quad \epsilon_{g,t} \sim N(0, \sigma_g^2),$$

where g is the steady-state ratio of government spending to output. Finally, the government balances the budget constraint every period given by

$$G_t = -T_t.$$

[2] The existence of state-contingent securities ensures that households are homogeneous with respect to consumption and asset holdings, even though they are heterogeneous with respect to the wage rate and hours because of the idiosyncratic nature of the timing of wage reoptimization. See Christiano et al. (2005).

2.6. Solution method

The choice of the solution method is very important. There are currently two major methods for solving DSGE models with seasonality. The first method is the one used in Chatterjee and Ravikumar (1992). We log-linearize the seasonal steady state around the balanced growth path and log-linearize the equilibrium conditions around the log-linearized seasonal steady state (CR method). The seasonal steady state is a periodic perfect foresight path that satisfies equilibrium conditions without uncertainty for each quarter. A more accurate alternative is the one used in Braun and Evans (1995). We directly solve for the seasonal steady state using a nonlinear solution method and log-linearize the equilibrium conditions around the exact seasonal steady state (BE method).

As is well known, a solution to a linear rational expectations system can be cast in a state-space representation. The state-space representation could form a basis of the Kalman filtering algorithm in building a likelihood for the estimation. The transition equation that characterizes the evolution of endogenous variables[3] is

$$\widehat{\mathbf{s}}_{t,q} = X_q(\theta)\widehat{\mathbf{s}}_{t-1,q-1} + Y_q(\theta)\epsilon_t,$$

where $\widehat{\mathbf{s}}_{t,q}$ is a vector that collects $\widehat{s}_{t,q} = \ln(s_{t,q}/s_q)$, which is the log-deviation of a variable $s_{t,q}$ in time t at quarter q from its seasonal steady state s_q. $X_q(\theta)$ and $Y_q(\theta)$ are the coefficient matrices that depend on a vector of the structural parameters θ, and ϵ_t is a vector of exogenous shocks. The CR method delivers a solution that restricts $X_q(\theta) = X(\theta)$ and $Y_q(\theta) = Y(\theta)$ for all quarters $q = 1, \dots, 4$. The BE method delivers a solution that allows $X_q(\theta)$ and $Y_q(\theta)$ to take different values across different quarters.

Now consider a seasonal adjustment procedure that subtracts the seasonal steady states from the data.[4] In this case we have $\widehat{\mathbf{s}}_t^{SA} = \widehat{\mathbf{s}}_{t,q}$, where $\widehat{\mathbf{s}}_t^{SA}$ is a vector that collects the log-deviations of the seasonally adjusted variables from their steady states. Suppose that an econometrician fits an aseasonal DSGE model to the seasonally adjusted data $\widehat{\mathbf{s}}_t^{SA}$. Observe that the CR method delivers consistent estimates. The BE method, on the other hand, may deliver important distortions, since the econometrician is fitting a model with constant $X(\theta)$ and $Y(\theta)$ to a data generating process where $X_q(\theta)$ and $Y_q(\theta)$ are periodically varying. Since my purpose is to quantify those distortions, I choose to work with the BE method.

3. The experiment

I ask whether using seasonally adjusted data leads to large distortions by estimating model parameters with simulated data from the seasonal DSGE model. First, I need to assign some values to the structural parameters and establish that the model is able to match certain features of U.S. seasonal and nonseasonal fluctuations.

3.1. Parameterization

There are three sets of parameters. The first set of parameters are those that characterize technology, preferences, and the central bank policy in the model and do not vary over quarters. The second set of parameters are those that vary across quarters. The first and the second sets of parameters jointly determine the seasonal steady state. The third set of parameters are those that characterize the stochastic shock processes.

Table 1
Parameters that are fixed across quarters.

Parameter	Description	Value
Panel A: Technology, preference, policy		
g	SS government spending	0.19
δ_0	SS depreciation rate	0.025
δ_2	Curvature of utilization cost	0.1
γ	SS technology growth	1.003
π	SS inflation rate	1.011
β	Discount factor	0.998
$\frac{\theta_p}{\theta_p-1}-1$	SS price markup	0.1
$\frac{\theta_w}{\theta_w-1}-1$	SS wage markup	0.1
α	Capital share	0.3
b	Habit persistence	0.7
η	Inverse Frisch elasticity	2
κ	Investment adjustment cost	1
ξ_p	Calvo price	0.6
ξ_w	Calvo wage	0.6
χ_p	Price indexation	0.3
χ_w	Wage indexation	0.3
ρ_R	Taylor rule smoothing	0.7
ϕ_π	Taylor rule inflation	1.7
ϕ_Y	Taylor rule output	0.2
Panel B: Shock process		
ρ_z	Neutral technology	0.95
ρ_τ	Preference	0.95
ρ_μ	Investment technology	0.95
ρ_π	Inflation target	0.95
ρ_g	Government spending	0.95
$100\sigma_z$	Neutral technology	0.9
$100\sigma_\tau$	Preference	1.7
$100\sigma_\mu$	Investment technology	1.4
$100\sigma_\pi$	Inflation target	0.1
$100\sigma_R$	Monetary policy	0.1
$100\sigma_g$	Government spending	1

The first set of parameters are reported in Panel A in Table 1. The parameters are picked around the values typically calibrated or estimated in the literature. The only parameter that deserves further attention is the parameter that controls κ (investment adjustment cost). The value ($\kappa = 1$) is slightly smaller than the values usually found in the literature. I assign this value because for larger adjustment costs, I had to assume implausibly large seasonal shifts in μ (investment technology) to match the seasonal pattern of investment observed in the U.S. data.

The second set of parameters are reported in Table 2. I allow the steady-state values of z (neutral technology), τ (preference), and μ (investment technology) to vary across quarters. Using a numerical minimization routine and a nonlinear equation solver, I calibrate the values so that the seasonal patterns of output, investment, and hours worked in the model match those in the data. Note that the average value of each parameter over quarters is ensured to be unity. Table 3 compares seasonal patterns in the data and in the model, given the assigned values of the first and second set of parameters. The model fit is very good. In particular, the model correctly predicts that seasonality is small in nominal variables such as the growth rate of real wages and the inflation rate.[5] There are two reasons for this success. First, prices and wages are assumed to be sufficiently sticky. This makes prices and wages less responsive to seasonal shifts in z, τ, and μ. In fact, if I lower the Calvo price and wage parameters (which implies less price and wage stickiness), I find that the seasonal steady states of the growth rate of wages and inflation become considerably more

3 For simplicity, I assume that all endogenous variables are observable to an econometrician (i.e., the coefficient matrix of the observation equation is an identity matrix with no measurement error). All of the discussion below extends to the more general case where some variables are latent.

4 Note that any reasonable seasonal adjustment filter can accomplish this task.

5 I fix the steady state of the nominal interest rate to be constant across quarters.

Table 2
Parameters that vary across quarters.

Parameter	Description	Q1	Q2	Q3	Q4	Average
z_q	Neutral technology	0.99	1.00	0.99	1.02	1.00
τ_q	Preference	0.85	1.04	1.01	1.10	1.00
μ_q	Investment technology	0.88	1.06	0.98	1.08	1.00

volatile over seasons. Second, seasonal shifts in τ (preference) effectively dampen the seasonal fluctuations in the real interest rate. To understand this, consider the intertemporal Euler equation as in Liu (2000):

$$1 = \beta \mathrm{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\frac{R_t}{\pi_{t+1}} \right) \right],$$

where λ_t denotes the marginal utility of consumption in period t. Suppose for a moment $b = 0$ and $\tau_t = 1$ for all quarters. Then $\lambda_{t+1}/\lambda_t = C_t/C_{t+1}$. Given the strong seasonality in consumption observed in the U.S. data, the real interest rate also has to exhibit strong seasonality in order to cancel out shifts in λ_{t+1}/λ_t. Seasonal fluctuations in τ perform a role of seasonal adjustment in λ_{t+1}/λ_t so that the interest rate becomes relatively stable across seasons.[6]

Finally, the third set of parameters are reported in Panel B of Table 1. The parameters are chosen so that second moments of the model resemble those of the data. In the Technical Appendix (available from the author's website), by comparing moments I show that overall the model is successful in replicating business cycle features of U.S. aggregate data. I conclude that the model serves as an empirically credible data generating mechanism for exploring the effects of estimating DSGE models using seasonally adjusted data.

3.2. Estimation using simulated data

Given the parameterization described above, I simulate 200 observations of artificial data sets (after throwing away the initial 100 periods). I employ a Bayesian procedure. The likelihood is calculated based on the following vector of observables:

$$[\Delta \ln Y_t, \Delta \ln C_t, \Delta \ln H_t, \Delta \ln(W_t/P_t), \ln \pi_t, \ln R_t],$$

where Δ is the first-difference operator. I conduct two different estimation experiments. In the first experiment, I estimate the baseline seasonal DSGE model using seasonally unadjusted data. In the second experiment, I estimate the aseasonal version of the baseline DSGE model using seasonally adjusted data. Specifically, during estimation I impose $z_q = 1, \tau_q = 1$, and $\mu_q = 1$ for all quarters $q = 1, \dots, 4$. All data except for interest rates are seasonally adjusted using the X-12-Arima filter.[7]

During estimation, I fix g, δ_0, and δ_2 to the true value, since they are difficult to identify. I also fix γ, π, and β, since most information for those parameters is contained in the levels of data and should not be affected much by the seasonal adjustment. Finally, I fix the steady-state price and wage markup, since I ran into some numerical difficulties when exploring the posterior distributions.[8] I assume flat priors for all the parameters, subject to some loose boundary constraints. As pointed out in, e.g., Fernández-Villaverde and Rubio-Ramírez (2008), with flat priors the posterior is proportional to the likelihood function. Thus the mode of the posterior can be interpreted as the parameter estimates of a maximum likelihood exercise.

4. Results

The estimates of the posterior distributions based on 200,000 draws from a random-walk Metropolis–Hastings algorithm are presented in Table 4. There are three things to observe. First, using seasonally unadjusted data delivers estimates that are quite close to the true values. This is not surprising, as I am estimating a correctly specified model using unfiltered data. Second, using X-12-Arima-filtered data delivers distorted estimates compared to using unadjusted data. The distortions are pronounced in some of the key structural parameters, such as α (capital share), η (inverse of the Frisch labor supply elasticity), κ (investment adjustment cost), ξ_w (Calvo wage parameter), and ϕ_π (Taylor rule coefficient on inflation). Third, the standard deviations of posterior distributions are smaller when seasonally unadjusted data are used. As discussed in Barsky and Miron (1989), seasonal fluctuations provide additional identifying restrictions that are not present in nonseasonal fluctuations, and hence I am able to obtain sharper estimates.

In Fig. 1, I report the log-likelihood profiles for a selected set of parameters given seasonally unadjusted data (solid lines) and X-12-Arima-filtered data (dashed lines).[9] I move each structural parameter around its calibrated value in each panel while fixing other parameters at their calibrated values. To facilitate comparison, I show the true value for each parameter in a vertical line. Information drawn from Fig. 1 is similar to that drawn from Table 4. The seasonally unadjusted likelihood peaks around the true parameter values, while the seasonally adjusted likelihood delivers considerable biases for many structural parameters. Directions of the biases are similar to those reported in Table 4.

Some readers may think that my results are sensitive to the way I seasonally adjust the simulated data. To ensure the robustness of the results against different choices of seasonal adjustment filters, I seasonally adjust the data using two alternative methods. First, I seasonally adjust using the Tramo–Seats filter.[10] Second, I seasonally adjust by directly using the DSGE model.[11] To understand the second procedure, recall that the law of motion for endogenous variables is given by

$$\widehat{\mathbf{s}}_{t,q} = X_q(\theta)\widehat{\mathbf{s}}_{t-1,q-1} + Y_q(\theta)\epsilon_t,$$

where $\widehat{\mathbf{s}}_{t,q}$ is a vector that collects $\widehat{s}_{t,q} = \ln(s_{t,q}/s_q)$. This can be rewritten as $s_{t,q} = s_q \exp(\widehat{s}_{t,q})$. To seasonally adjust $s_{t,q}$, replace a seasonal steady state from the seasonal model, s_q, with a steady

6 The first-order condition for each household's labor supply indicates that the marginal utility of consumption is also connected to movements in real wages and hours. While strong seasonality in hours observed in the data may suggest that wages also have to display strong seasonality in order to compensate for the weak seasonality in the marginal utility, this is not necessarily the case in our environment. In fact, the optimal wage for households adjusting their individual wages is relatively constant across seasons, since wages are sticky and hence households care about the influence of their current wage choice on their labor supply not only in the current quarter but also in future quarters. In other words, wage-setting policies that respond to seasonal movements of hours only in the current quarter are sub-optimal.

7 X-12-Arima is a software package developed by the U.S. Census Bureau and is the official seasonal adjustment procedure of the U.S. government. The seasonal adjustment is conducted using software called "Demetra", which is provided by Eurostat.

8 More specifically, the problem arises when I estimate the model using seasonally adjusted data. I also re-estimated the model by fixing the price and wage markup at several other values and found that the qualitative features of the results are unaffected.

9 The log-likelihood profiles for other parameters are given in the Technical Appendix.

10 Tramo–Seats is a time series analysis package constructed from signal extraction principles and used extensively at the European Central Bank and Eurostat. The Bank of Spain's website (http://www.bde.es/servicio/software/econome.htm) provides a detailed explanation of the procedure.

11 I thank the Associate Editor for the suggestion.

Table 3
Seasonal patterns.

Series	Data				Model			
	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Output*	−6.37	3.08	0.61	2.69	−6.37	3.08	0.61	2.69
Consumption	−6.50	2.27	0.09	4.14	−5.59	2.07	0.58	2.94
Investment*	−8.12	5.30	0.68	2.14	−8.11	5.30	0.68	2.14
Hours*	−3.83	3.00	1.59	−0.59	−3.87	2.97	1.55	−0.64
Wage growth	−0.45	−0.53	0.36	0.61	−0.53	0.17	0.06	0.30
Inflation rate	0.09	0.23	−0.14	−0.18	0.40	0.01	−0.06	−0.35
Interest rate	−0.03	0.04	0.01	−0.03	0.00	0.00	0.00	0.00

Notes: the table reports percent changes of variables from the previous quarter, taken from sample averages in the data and seasonal steady states in the model.
* Indicate those used as calibration targets.

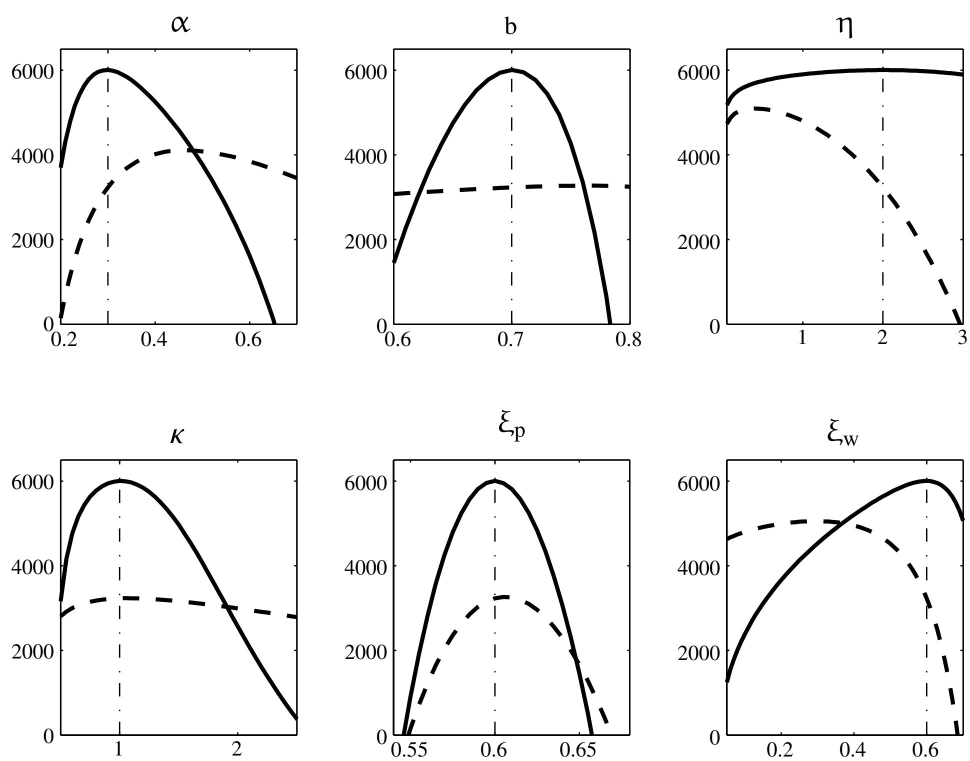

Fig. 1. Likelihood profiles: seasonally adjusted vs. unadjusted data. *Notes*: the figure plots likelihood profiles for seasonally unadjusted data (solid lines) and X-12-Arima-filtered data (dashed lines). Vertical lines signify true values.

state from the aseasonal model, s: $s_{t,q} = s\exp(\widehat{s}_{t,q})$. Thus, the procedure can be thought of as regressing the data on seasonal dummies, but in a way consistent with the DSGE model. I simply call this the "DSGE-based" seasonal adjustment. As I show in the Technical Appendix, for both methods, the posterior estimates are very similar compared to when X-12-Arima-filtered data are used.

I argue that these distortions in parameter estimates are important for economic inference because they (1) alter the transmission mechanism of shocks, (2) affect the business cycle statistics generated by the model, and (3) bias the results of policy analysis.

To illustrate the first point, I compare the impulse responses based on seasonally adjusted and unadjusted estimates. Comparing impulse responses is tricky here, since when seasonality is present the transmission of shocks differs considerably when the quarter in which the shock hits is different. Thus I consider the following comparison.

1. From the seasonal DSGE model, I draw parameters from the posterior distribution of seasonally unadjusted estimates and compute impulse responses. To compute impulse responses, I first compute four versions of impulse responses, each differing with respect to the quarter in which the shock hits. I then take the average of the four responses. The resulting response could be thought of as a "seasonally adjusted" impulse response (i.e., impulse response without conditioning on a season) of the seasonal model.
2. From the aseasonal DSGE model, I draw parameters from the posterior distribution of X-12-Arima-filtered estimates and compute impulse responses.

I note that the "seasonally adjusted" responses generated from the seasonal model are almost identical to the responses generated from the aseasonal model when the same parameter values are used. Comparing the two versions of impulse responses, I can ask whether the impulse responses using the seasonally adjusted estimates can successfully predict the average response across quarters. I plot mean posterior impulse responses and their 90% point-wise intervals of a neutral technology shock and a monetary policy shock in Figs. 2 and 3, respectively. Observe that the true responses generated from the seasonal DSGE model are very close to the mean responses of the seasonally unadjusted estimates. The responses are qualitatively similar between the seasonally adjusted and unadjusted estimates. For example, an exogenous improvement in technology robustly delivers hump-shaped increases in output and investment, persistent increases

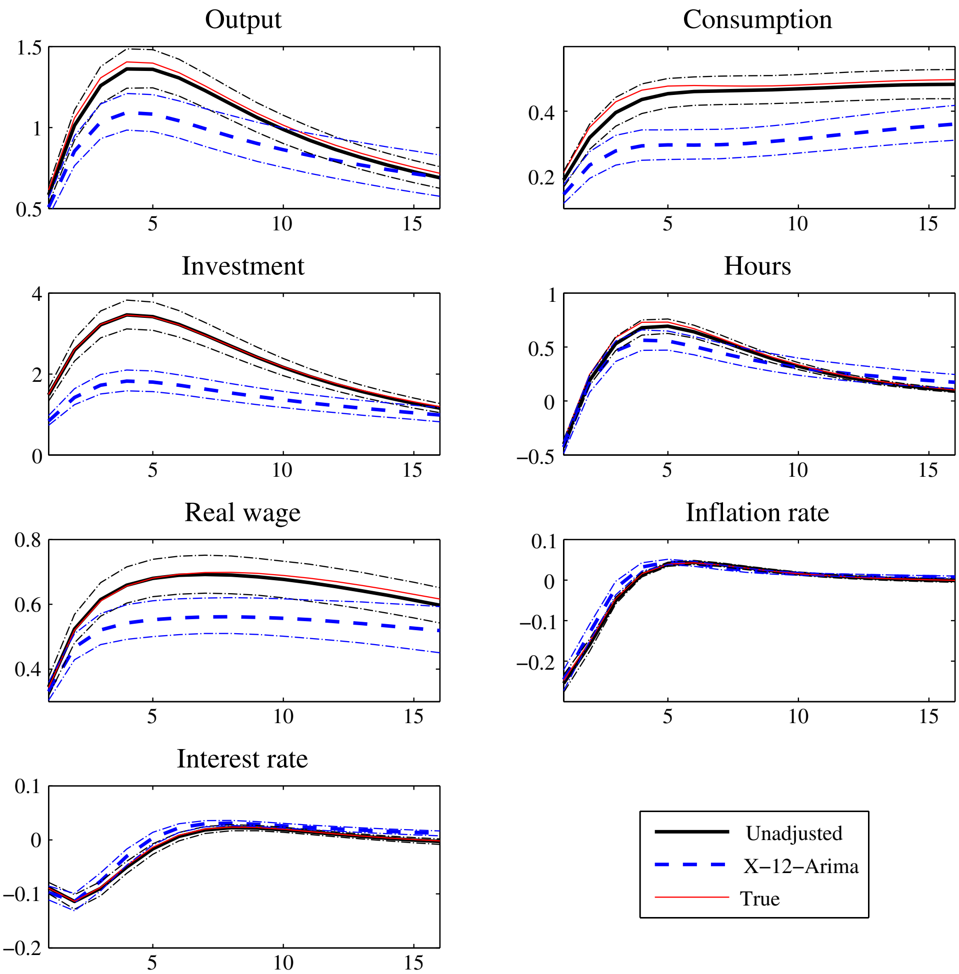

Fig. 2. Impulse response: neutral technology shock. *Notes*: each panel describes the percentage-point response to a one-standard-deviation shock. Computations are based on 1000 draws from the posterior distributions.

in consumption and real wages, and immediate declines in hours worked and inflation. An exogenous decrease in the interest rate leads to moderate but persistent increases in output, consumption, investment, hours, and inflation. Note, however, that there are also some important quantitative differences. For example, the seasonally adjusted estimates considerably understate output, consumption, and investment responses to an improvement in technology. Interestingly, the responses of hours worked are precisely matched. They also understate output, consumption, and investment responses to an expansionary monetary policy shock, but again the responses to hours worked are precisely matched. On the other hand, inflation responses to a monetary policy shock are overstated.

To substantiate the second point, I compare the second moments generated from two sources. To generate a first set of moments, I simulate data from the seasonal model with parameters fixed at the posterior means of seasonally unadjusted estimates and then seasonally adjust the data using the X-12-Arima filter. To generate a second set of moments, I simulate data from the aseasonal model with parameters fixed at the means of the X-12-Arima-filtered estimates. In Table 5, I compare those two sets of moments, together with the moments generated from the seasonal model under the true parameters.[12] Columns under the label "Percent standard deviation" in Table 5 show that the seasonally adjusted estimates considerably understate the standard deviation of output growth and overstate the standard deviation of hours growth, both by about 0.10. Moreover, they predict only about half the volatility of investment growth. Columns under the label "Corr. with output growth" in Table 5 show that correlations with output growth are in general understated. For example, using seasonally adjusted estimates, consumption growth correlation is less than half of what is predicted using the true parameters or seasonally unadjusted estimates. In evaluating those differences in moments, it is important to note that when the second set of moments are generated from the aseasonal model using seasonally unadjusted estimates (instead of seasonally adjusted estimates), the two sets of moments are almost identical and close to the true moments.

Finally I show that bias in point estimates translates into bias in policy analysis. I consider the following counterfactual policy experiment.[13] I compute the percent standard deviations of output growth and inflation when I increase the inflation response coefficient in the Taylor rule from the benchmark value ($\phi_\pi = 1.7$), both for the seasonal model using seasonally adjusted estimates and the aseasonal model using X-12-Arima-filtered estimates. Again,

12 Fernández-Villaverde and Rubio-Ramírez (2005) document that moments generated from linear and nonlinear likelihood estimates are considerably different.

13 For other work on policy experiments in misspecified DSGE models, see Chang et al. (forthcoming) and Cogley and Yagihashi (2010).

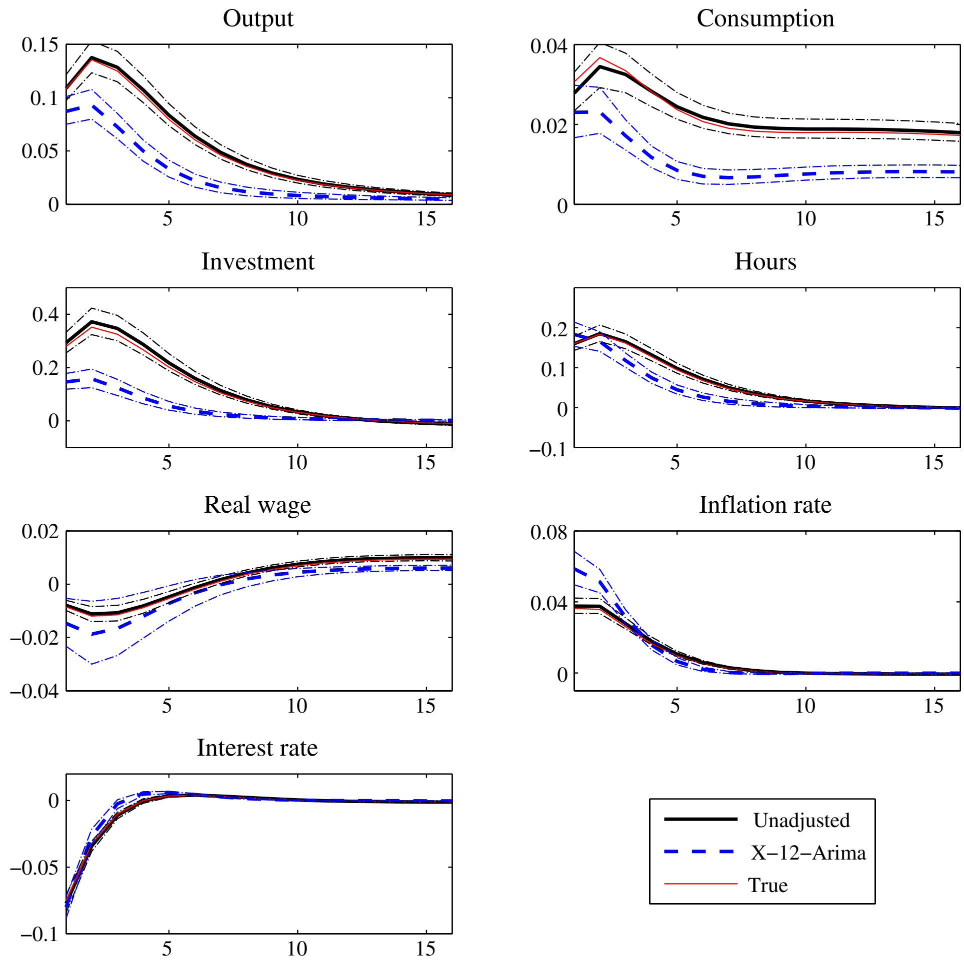

Fig. 3. Impulse response: monetary policy shock. *Notes*: each panel describes the percentage-point response to a one-standard-deviation shock. Computations are based on 1000 draws from the posterior distributions.

simulated data from the seasonal model are adjusted using the X-12-Arima filter. The results are shown in Table 6. The seasonally unadjusted estimates correctly predict the size of changes in output growth and inflation volatilities in response to the increase in ϕ_π. The seasonally adjusted estimates correctly predict the changes in inflation volatility. However, they understate the magnitude of the increase in output growth volatility. While both true parameters and seasonally unadjusted estimates predict that the standard deviation of output growth increases by about 30% compared to the benchmark case when $\phi_\pi = 10$, the seasonally adjusted estimates predict that it increases by only about 10%.

The results presented so far are important for applied macroeconomics research. They suggest that the conventional practice of estimating DSGE models using seasonally adjusted data may lead to biased inference, and hence policy experiments based on the estimated parameters could be misleading.[14]

5. Inspecting the sources of distortions

Why does estimating the DSGE model using seasonally adjusted data create sizable distortions, as reported in the previous section? In the first subsection, I show that the main reason for the distortions is that the effects of seasonality are not restricted to the seasonal frequencies, but instead are propagated across the entire frequency domain. In the second subsection, I argue that the capital accumulation and labor market frictions in the model amplify nonlinear interactions between seasonality and endogenous variables and make the distortions quantitatively relevant.

5.1. Evidence from the frequency domain

Before turning to a detailed investigation, first it would be useful to take a look at what the standard seasonal adjustment methods do to the data. In Fig. 4, I plot the sample periodogram of the simulated data (seasonally unadjusted, X-12-Arima-filtered, Tramo–Seats-filtered, and DSGE-based-filtered data) used in the previous section.[15] First, the spectra of seasonally unadjusted data have spikes at seasonal frequencies $\left(\omega = \pi\right.$ and, in particular, $\left.\frac{\pi}{2}\right)$. Second, the seasonal adjustment procedures eliminate those seasonal spikes but leave the spectral densities at other frequencies unaffected. These observations suggest that the distortions found in the previous section are due to the fact that the seasonal

[14] I also conducted experiments replacing $\Delta \ln I_t$ with $\Delta \ln C_t$ as observables. In this case, using seasonally adjusted data still leads to substantially distorted estimates. However, using seasonally unadjusted data, the parameters controlling the government shock process (ρ_g and $100\sigma_g$) are imprecisely estimated due to a weak identification problem. For this reason, I focus on results that use $\Delta \ln C_t$ as observables for the rest of the paper.

[15] The periodogram is smoothed by taking the equally weighted average of periodograms on 7 frequencies at and in the neighborhood of each frequency $\omega_j = 2\pi j/T, j = 0, 1, \ldots, T-1$.

Table 4
Posterior estimates.

Parameter	Description	True	Unadjusted	X-12
α	Capital share	0.3	0.29 (0.0120)	0.50 (0.0431)
b	Habit persistence	0.7	0.73 (0.0134)	0.71 (0.0270)
η	Inverse Frisch elasticity	2	2.01 (0.1287)	1.01 (0.1646)
κ	Investment adjustment cost	1	1.03 (0.0760)	1.49 (0.1905)
ξ_p	Calvo price	0.6	0.60 (0.0024)	0.57 (0.0084)
ξ_w	Calvo wage	0.6	0.59 (0.0061)	0.49 (0.0321)
χ_p	Price indexation	0.3	0.30 (0.0080)	0.33 (0.0311)
χ_w	Wage indexation	0.3	0.30 (0.0112)	0.35 (0.0409)
ρ_R	Taylor rule smoothing	0.7	0.71 (0.0178)	0.72 (0.0302)
ϕ_π	Taylor rule inflation	1.7	1.69 (0.1398)	2.11 (0.3506)
ϕ_Y	Taylor rule output	0.2	0.22 (0.0499)	0.34 (0.1047)
ρ_z	Neutral technology	0.95	0.95 (0.0020)	0.95 (0.0059)
ρ_τ	Preference	0.95	0.94 (0.0091)	0.97 (0.0076)
ρ_μ	Investment technology	0.95	0.96 (0.0103)	0.92 (0.0222)
ρ_π	Inflation target	0.95	0.96 (0.0066)	0.95 (0.0105)
ρ_g	Government spending	0.95	0.95 (0.0109)	0.62 (0.1210)
$100\sigma_z$	Neutral technology	0.9	0.90 (0.0452)	0.78 (0.0397)
$100\sigma_\tau$	Preference	1.7	1.72 (0.1010)	1.52 (0.1441)
$100\sigma_\mu$	Investment technology	1.4	1.68 (0.2194)	1.41 (0.1878)
$100\sigma_\pi$	Inflation target	0.1	0.09 (0.0120)	0.11 (0.0168)
$100\sigma_R$	Monetary policy	0.1	0.10 (0.0054)	0.12 (0.0072)
$100\sigma_g$	Government spending	1	0.90 (0.0472)	1.48 (0.1551)
$\tilde{z}_1$	Neutral technology Q1	0.97	0.97 (0.0005)	–
$\tilde{z}_2$	Neutral technology Q2	0.97	0.97 (0.0003)	–
$\tilde{z}_3$	Neutral technology Q3	0.97	0.97 (0.0001)	–
$\tilde{\tau}_1$	Preference Q1	0.77	0.74 (0.0126)	–
$\tilde{\tau}_2$	Preference Q2	0.95	0.94 (0.0023)	–
$\tilde{\tau}_3$	Preference Q3	0.92	0.92 (0.0037)	–
$\tilde{\mu}_1$	Investment technology Q1	0.81	0.80 (0.0121)	–
$\tilde{\mu}_2$	Investment technology Q2	0.98	0.97 (0.0034)	–
$\tilde{\mu}_3$	Investment technology Q3	0.91	0.91 (0.0069)	–

Notes: the table reports the MCMC estimates of posterior means. Standard deviations are reported in parentheses. The following reparameterizations are used: $\tilde{z}_q = z_q/z_4$, $\tilde{\tau}_q = \tau_q/\tau_4$, and $\tilde{\mu}_q = \mu_q/\mu_4$ for $q = 1, 2, 3$.

Table 5
Business cycle statistics: seasonally adjusted vs. unadjusted estimates.

Series	Percent standard deviation			Corr. with output growth		
	True	Unadjusted	X-12	True	Unadjusted	X-12
Output growth	0.96	0.94	0.83	–	–	–
Consumption growth	0.57	0.57	0.54	0.38	0.31	0.12
Investment growth	2.78	2.87	1.48	0.89	0.88	0.84
Hours growth	0.99	0.98	1.10	0.54	0.53	0.48
Wage growth	0.41	0.41	0.39	0.77	0.77	0.60
Inflation rate	0.73	0.78	0.70	−0.22	−0.21	−0.21
Interest rate	0.69	0.77	0.69	−0.18	−0.15	−0.16

Notes: true moments are calculated by applying the X-12-Arima filter to the simulated data from the seasonal model, where the parameters are fixed at their true values. Similarly, seasonally unadjusted moments are calculated by applying the X-12-Arima filter to the simulated data from the seasonal model, where the parameters are fixed at the posterior means of seasonally unadjusted estimates. X-12-Arima moments are calculated using simulated data from the aseasonal model, where the parameters are fixed at the posterior means of X-12-Arima-filtered estimates. I did not apply any seasonal adjustment filter to the simulated data for X-12-Arima moments. All simulations are based on 100 replications of artificial time series of length 200.

adjustment procedures fail to completely eliminate the effects of seasonality because seasonality also influences the spectral densities at other nonseasonal frequencies as well. For the rest of this section, I will formalize this argument by using a set of tools developed by previous authors.

In Fig. 5, for the seasonal and aseasonal DSGE models, I plot the log spectrum of the variables used in the estimation.[16] For both models the parameters are fixed at the values (reported in Tables 1 and 2) used to generate data for the experiment in the previous sections. The log spectrum of the seasonal model is shown in thick solid lines, and the log spectrum of the aseasonal model is shown in thick dashed lines. Observe that for output growth and hours growth, there are considerable discrepancies between the spectra of the seasonal and aseasonal model at the entire frequency domain. In particular, the discrepancies are noticeable at high frequencies (frequencies above $\omega = \frac{\pi}{2}$). In those regions, the seasonal model has more spectral power. On the other hand, for nominal variables such as wage growth, inflation rates, and interest rates, the discrepancies are small and confined to the seasonal frequencies $\left(\omega = \frac{\pi}{2}, \pi\right)$.[17]

The intuition behind these discrepancies is relatively straightforward. Since seasonality induces agents to reallocate their resources across seasons within a year, the discrepancies of spectra are noticeable at higher frequencies. Moreover, because of seasonality, agents have different responses across seasons to the same shocks, and this additional source of volatility raises the spectral power of the seasonal model.

Sims (1993) and Hansen and Sargent (1993) recommend using seasonally adjusted data in estimating rational expectations models. Their recommendation is based on two arguments. First,

Table 6
Percent standard deviation of variables under alternative values of ϕ_π: seasonally adjusted vs. unadjusted estimates.

Series	ϕ_π				
	1.7	2.5	5.0	7.5	10.0
Output growth					
True	0.96 [1.00]	1.00 [1.04]	1.12 [1.17]	1.21 [1.26]	1.27 [1.32]
Unadjusted	0.95 [1.00]	0.97 [1.03]	1.11 [1.17]	1.20 [1.26]	1.26 [1.33]
X-12	0.86 [1.00]	0.86 [1.00]	0.91 [1.06]	0.95 [1.10]	0.97 [1.12]
Inflation					
True	0.73 [1.00]	0.53 [0.73]	0.41 [0.56]	0.36 [0.50]	0.34 [0.47]
Unadjusted	0.77 [1.00]	0.55 [0.72]	0.42 [0.55]	0.37 [0.49]	0.35 [0.46]
X-12	0.84 [1.00]	0.62 [0.74]	0.45 [0.54]	0.42 [0.50]	0.39 [0.46]

Notes: other monetary policy parameters are set to the true benchmark values ($\rho_R = 0.7$, $\phi_Y = 0.2$). The numbers in square brackets indicate the ratios relative to the benchmark case ($\phi_\pi = 1.7$). For simulation details, see the footnote of Table 5.

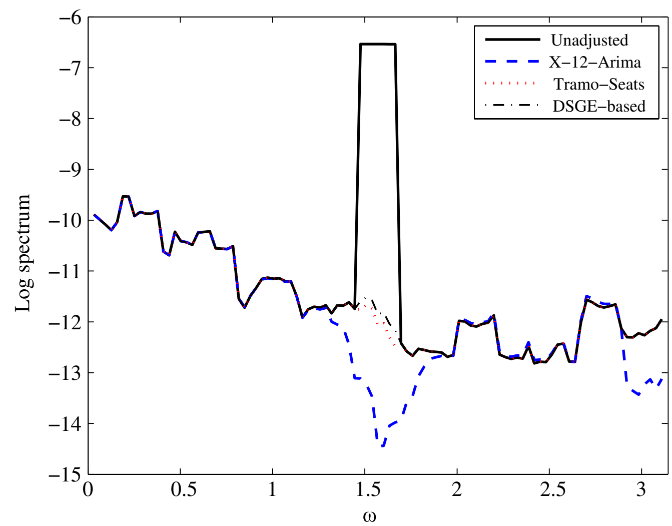

Fig. 4. Log spectrum of the simulated data (output growth).

[16] As in Hansen and Sargent (1993), I use the formula of Tiao and Grupe (1980) to compute the spectral densities of the seasonal model. The formula provides an expression for the mean-adjusted periodic process, without conditioning on a season of the year.

[17] The coherence shows a similar pattern of discrepancies. I omit the figures to conserve space.

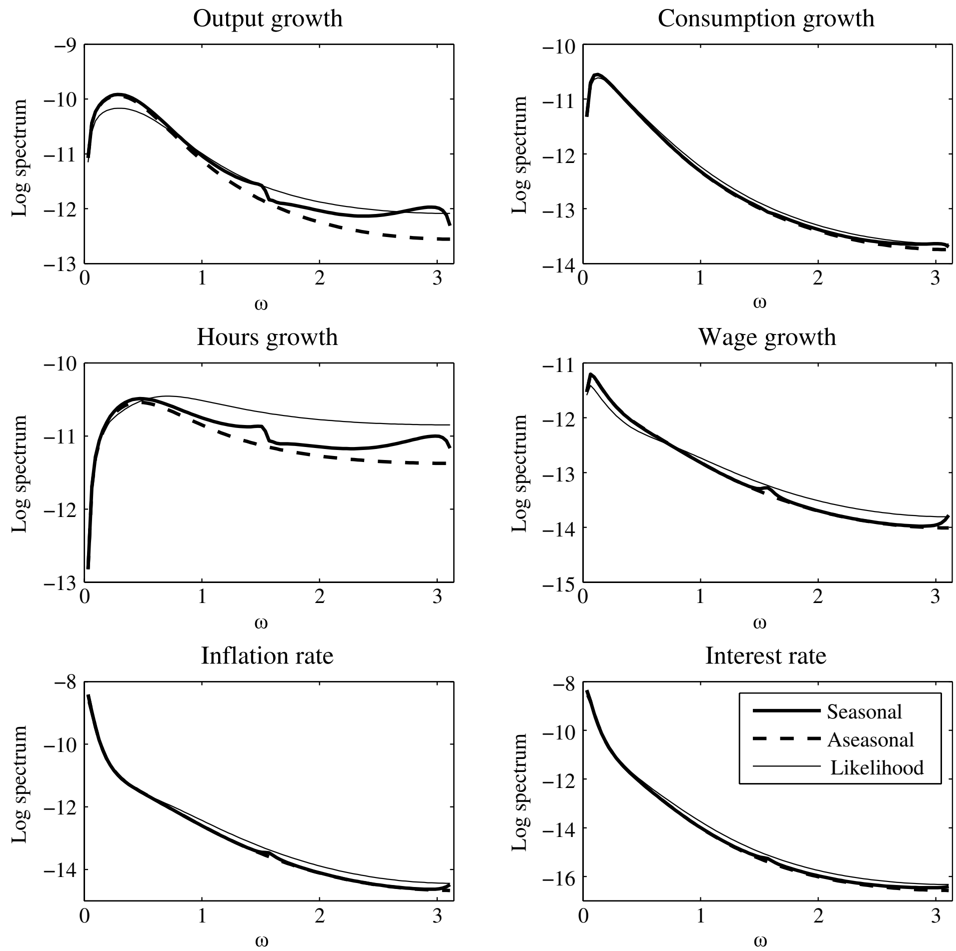

Fig. 5. Log spectrum of the seasonal and aseasonal model. *Notes*: thick solid and dashed lines plot spectra of seasonal and aseasonal models whose parameters are fixed at the values used to generate data in the main experiment. Thin solid lines plot spectra of the aseasonal model whose parameters are fixed at the asymptotic maximum likelihood estimates when seasonally adjusted data are used.

directly modeling seasonality may lead to large distortions, if the mechanism generating seasonality is misspecified. Second, since the effects of seasonality are likely to be confined to seasonal frequencies, dampening those seasonal frequencies by seasonally adjusting the data and trying to fit aseasonal models to the nonseasonal frequencies leads to fairly accurate estimates. In my model, the second argument does not hold. The effects of seasonality propagate across the entire frequency domain, and hence trying to fit the nonseasonal frequencies using the aseasonal version of the model leads to substantial distortions in parameter estimates.[18]

I consider the implications of the discrepancies of spectral densities by using a frequency domain approximation for the probability limits of misspecified maximum likelihood estimators developed by Hansen and Sargent (1993). The frequency domain approximation is useful for two reasons. First, it allows me to isolate the effects of discrepancies of spectral densities from other factors that potentially bias estimates (e.g., seasonal adjustment filters or weak identification due to small samples). Second, it allows me to take a closer look at which particular frequencies are responsible for the bias.

Hansen and Sargent (1993) show that the maximum likelihood estimator of a parameter vector θ converges almost surely to the minimizer of the following formula:

$$A(\theta) = A_1(\theta) + A_2(\theta) + A_3(\theta), \tag{1}$$

where

$$A_1(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \ln\det G(\omega;\theta)d\omega,$$

$$A_2(\theta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{trace}[G(\omega;\theta)^{-1}F(\omega)]d\omega,$$

$$A_3(\theta) = [\mu - \mu(\theta)]'G(0;\theta)^{-1}[\mu - \mu(\theta)].$$

μ is the population mean of a stationary process and $F(\omega)$ is the spectral density function at frequency ω. $\mu(\theta)$ and $G(\omega;\theta)$ are the model-based mean and spectral density function. $A_1(\theta)$ captures the variance of the model-based one-step forecast errors. $A_2(\theta)$ and $A_3(\theta)$ measure the distance between data and model-based spectral densities and means, respectively. In implementing expression (1), it is useful to approximate the integrals in $A_1(\theta)$ and $A_2(\theta)$ by Riemann sums:

$$\widehat{A}_1(\theta) = \frac{1}{T}\sum_{j=0}^{T-1} \ln\det G(\omega_j;\theta),$$

$$\widehat{A}_2(\theta) = \frac{1}{T}\sum_{j=0}^{T-1} \mathrm{trace}[G(\omega_j;\theta)^{-1}F(\omega_j)],$$

where $\omega_j = 2\pi j/T, j = 0, 1, \ldots, T-1$.

[18] Ghysels (1988) presents a simple production market model demonstrating this phenomenon. Also see Cogley (2001), Canova (2009), and Canova and Ferrroni (2011) for a related point concerning the interactions of trend and cyclical components in DSGE models.

The column labeled "Baseline" in Table 7 reports the probability limits of maximum likelihood estimates when seasonally adjusted data are used. To compute the probability limits, I apply the Riemann sum approximation of formula (1), with $T = 200$ and $G(\omega;\theta)$ generated from the aseasonal DSGE model and $F(\omega)$ generated from the seasonal DSGE model. Zero weight is assigned to frequencies at and near the seasonal frequencies[19] and deterministic seasonal means are removed by dropping $A_3(\theta)$ from formula (1). This allows me to mimic an "ideal" seasonal adjustment procedure in the frequency domain.

The biases in parameter estimates are similar to those reported in Table 4, although their magnitudes are slightly smaller. In Fig. 5, the spectral density of the asymptotic maximum likelihood estimates is plotted in solid lines. In order to achieve better fit in output and hours growth, the likelihood estimator tries to shift spectral power from low to high frequencies by distorting the parameter estimates.[20]

Another useful measure to examine is a version of the likelihood-ratio statistic developed in Christiano and Vigfusson (2003):

$$\lambda = 2[A(\theta_{true}) - A(\theta^*)],$$

where θ_{true} is a vector of parameters fixed at their true values and θ^* is a vector of parameters fixed at their estimated values (in this case asymptotic maximum likelihood estimates). Define

$$\lambda(\omega) = \ln\det G(\omega;\theta_{true}) - \ln\det G(\omega;\theta^*) + \mathrm{trace}[(G(\omega;\theta_{true})^{-1} - G(\omega;\theta^*)^{-1})F(\omega)],$$

so that

$$\lambda = \lambda(0) + 2\sum_{j=1}^{T/2-1}\lambda(\omega_j) + \lambda(\pi).$$

The cumulative likelihood ratio is defined as

$$\Lambda(\omega) = \lambda(0) + 2\sum_{\omega_j\le\omega}\lambda(\omega_j), \quad 0<\omega<\pi,$$

$$\Lambda(0) = \lambda(0),$$

$$\Lambda(\pi) = \lambda.$$

If bias of the estimated parameters is due to discrepancies of seasonal and aseasonal spectra in some specific frequency region, we should see a sharp increase in $\Lambda(\omega)$. Fig. 6 shows that there is a sharp increase at medium and high frequencies.[21] On the other hand, there is a mild decrease in the ratio at low frequencies. This observation confirms that the likelihood estimator is distorting the estimated parameters in order to achieve a better fit at higher frequencies.

5.2. The role of frictions

My model features a number of real and nominal frictions. The frictions magnify the nonlinear interactions between seasonality and endogenous variables, which in turn leads to larger discrepancies of spectra between the seasonal and aseasonal DSGE models. Thus, to understand the source of bias, it is crucial to know the quantitative role of each friction in the model. To this end, I turn off each friction of the model, recompute the probability limits, and compare the resulting biases of the estimates with the baseline model. I identify two key frictions – the investment adjustment cost and the nominal wage rigidity – which play important roles.

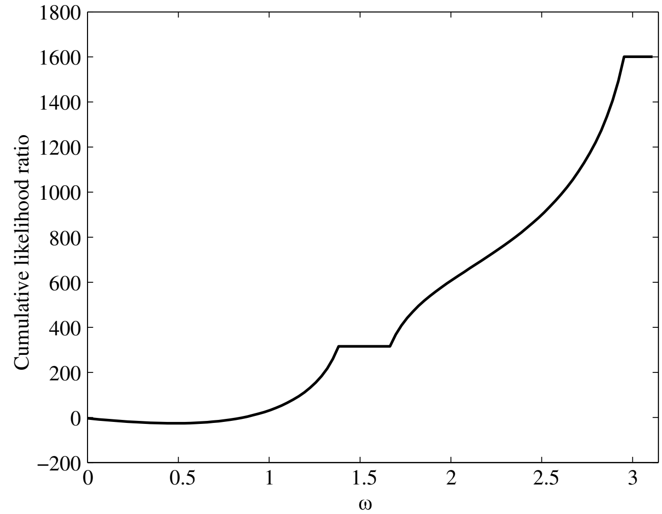

Fig. 6. Cumulative likelihood ratio.

In the column in Table 7 labeled "No inv. adj.", I report the probability limits of the maximum likelihood estimator when the investment adjustment cost is turned off ($\kappa = 0$). Since the magnitude of the adjustment cost does affect the seasonal steady states, I recalibrate seasonal shifts in neutral and investment technology and preference in order to match the data. I also adjust the parameters characterizing stochastic shock processes so that the model without the adjustment cost generates realistic second moments.[22] The estimated parameters come closer to the true values compared to those reported in the column labeled "Baseline". In particular, for some key parameters, including α (capital share), η (inverse of the Frisch labor supply elasticity), ξ_w (Calvo wage parameter), and ϕ_π (Taylor rule coefficient on inflation), distortions disappear almost completely. In the column labeled "No wage rig.", I report the probability limits when the wage rigidity is (almost) turned off ($\xi_w = 0.01, \chi_w = 0$). Again, I recalibrate the seasonal shifts in technology and preference and readjust the parameters characterizing the stochastic shock processes. Similar to the case when the investment adjustment cost is turned off, the distortions are quite small (except for the persistence parameter of government spending, ρ_g, which is considerably understated). I have also examined model specifications where habit persistence is turned off ($b = 0$), capital utilization is turned off ($\delta_2 = 1000$), price rigidity is turned off ($\xi_p = 0.01, \chi_p = 0$), price and wage indexation is turned off ($\chi_p = 0, \chi_w = 0$), and the Taylor rule responding to a deviation of output (rather than output growth) from the steady state. None of these alternative specifications delivered precise estimates.[23] I view these as evidence showing

[19] I impose zero weight to the 9 frequencies at and in the neighborhood of $\omega = \pi/2$, and also $\omega = \pi$ and the 4 next lower frequencies. The results are robust to the choice of the number of frequencies assigned zero weight.

[20] As pointed out in Cogley (2001), it is difficult to develop intuition of a direction of the bias in a particular parameter when all parameters are allowed to adjust simultaneously. This is because sometimes the partial effects of parameter adjustments interact in ways that counteract one another. For example, while the upward bias in α (capital share) and the downward bias in η (inverse of the Frisch labor supply elasticity) act in ways that raise the spectral power of output and hours growth, the upward bias in κ (investment adjustment cost) dampens it.

[21] Note that since I omit the seasonal frequencies and their neighborhood during computation of the probability limits, the cumulative likelihood ratio is flat in that region.

[22] Without adjustment, the volatilities of output and hours growth become extremely large. Moreover the spectra of those variables reach their peak at the highest frequency, which is the opposite of what we see in the data (Granger, 1966). As pointed out by Christiano and Todd (2002), since the weight assigned in the approximation criterion to the spectra is proportional to the level of the corresponding empirical estimates (expression (1)), even a very small discrepancy in the spectrum at higher frequencies creates implausibly large parameter biases.

[23] Details of the results are available from the author upon request.

Table 7
Probability limits of the likelihood estimator when seasonally adjusted data are used.

Parameter	Description	Baseline		No inv. adj.		No wage rig.		Capital adj.		Labor adj.	
		True	Plim	True	Plim	True	Plim	True	Plim	True	Plim
α	Capital share	0.3	0.46	0.3	0.31	0.3	0.27	0.3	0.39	0.3	0.26
b	Habit persistence	0.7	0.67	0.7	0.70	0.7	0.71	0.7	0.68	0.7	0.70
η	Inverse Frisch elasticity	2	1.24	2	2.15	2	1.99	2	2.07	2	1.94
κ	Investment adjustment cost	1	1.25	–	–	1	0.89	–	–	1	0.77
κ_K	Capital adjustment cost	–	–	–	–	–	–	24	43.45	–	–
κ_H	Labor adjustment cost	–	–	–	–	–	–	–	–	0.8	0.80
ξ_p	Calvo price	0.6	0.57	0.6	0.60	0.6	0.60	0.6	0.58	0.6	0.63
ξ_w	Calvo wage	0.6	0.54	0.6	0.60	–	–	0.6	0.60	–	–
χ_p	Price indexation	0.3	0.31	0.3	0.29	0.3	0.29	0.3	0.32	0.3	0.19
χ_w	Wage indexation	0.3	0.32	0.3	0.31	–	–	0.3	0.32	–	–
ρ_R	Taylor rule smoothing	0.7	0.71	0.7	0.70	0.7	0.70	0.7	0.72	0.7	0.66
ϕ_π	Taylor rule inflation	1.7	1.81	1.7	1.72	1.7	1.72	1.7	1.77	1.7	1.45
ϕ_Y	Taylor rule output	0.2	0.22	0.2	0.20	0.2	0.21	0.2	0.21	0.2	0.10
ρ_z	Neutral technology	0.95	0.95	0.95	0.95	0.95	0.95	0.95	0.46	0.95	0.95
ρ_τ	Preference	0.95	0.96	0.95	0.95	0.95	0.95	0.95	0.96	0.95	0.94
ρ_μ	Investment technology	0.95	0.92	0.95	0.79	0.95	0.96	0.95	0.86	0.95	0.96
ρ_π	Inflation target	0.95	0.94	0.95	0.96	0.95	0.95	0.95	0.95	0.95	0.96
ρ_g	Government spending	0.95	0.86	0.95	0.92	0.95	0.42	0.95	0.92	0.95	0.28
$100\sigma_z$	Neutral technology	0.9	0.87	0.4	0.40	1.3	1.32	0.9	0.18	1.2	1.22
$100\sigma_\tau$	Preference	1.7	1.51	2	2.03	1.6	1.70	1.7	1.71	1.2	1.23
$100\sigma_\mu$	Investment technology	1.4	1.32	0.1	0.05	1.4	1.69	1.4	9.78	2	2.37
$100\sigma_\pi$	Inflation target	0.1	0.10	0.01	0.01	0.08	0.09	0.1	0.11	0.06	0.08
$100\sigma_R$	Monetary policy	0.1	0.11	0.01	0.01	0.1	0.10	0.1	0.10	0.1	0.10
$100\sigma_g$	Government spending	1	1.94	1	1.68	1	1.38	1	1.39	1	1.36

that the investment adjustment cost and the nominal wage rigidity are the key frictions responsible for creating distortions.

Given the finding, readers might guess that other forms of capital accumulation or labor market frictions may contribute to creating distortions as well. This is indeed the case. To formalize the argument, I consider two alternative model specifications where (a) the investment adjustment cost is replaced with a capital adjustment cost and (b) the sticky wage assumption is replaced with a labor adjustment cost, and see whether the seasonal adjustment creates distortions.

For the capital adjustment cost, consider

$$K_t^p = (1-\delta(u_t))K_{t-1}^p + \mu_t\left(I_t - S\left(\frac{K_t^p}{K_{t-1}^p}\right)K_{t-1}^p\right),$$

where for the functional form for S, I assume

$$S\left(\frac{K_t^p}{K_{t-1}^p}\right) = \frac{\kappa_K}{2}\left(\frac{K_t^p}{K_{t-1}^p} - \gamma\right)^2.$$

Similar specifications for the capital adjustment cost were used, for example, in Bernanke et al. (1999) and Chari et al. (2000). I set $\kappa_K = 24$ so that the moments generated from the capital adjustment cost model are similar to those generated from the baseline model. The column in Table 7 labeled "Capital adj". reports the probability limits of the maximum likelihood estimator. As in the baseline model, the capital adjustment cost model delivers sizable distortions, although their directions and magnitudes are somewhat different. For example, α (capital share), κ (capital adjustment cost), and $100\sigma_\mu$ (volatility parameter of the investment technology shock process) are overstated. Also ρ_z and $100\sigma_z$ (persistence and volatility parameters of the neutral technology shock process) are understated.

To investigate the role of the labor adjustment cost, I simply add a quadratic disutility term into the household's utility function:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \left[\tau_{t+s}\ln(C_{t+s} - bC_{t+s-1}) - \varphi\frac{H_{i,t+s}^{1+\eta}}{1+\eta} - \frac{\kappa_H}{2}\left(\frac{H_{i,t+s}}{H_{i,t+s-1}} - 1\right)^2\right].$$

I impose $\kappa_H = 0.8$ and recalibrate the seasonal shifts in technology and preference and readjust the parameters characterizing the stochastic shock processes. The column in Table 7 labeled "Labor adj". reports the probability limits of the maximum likelihood estimator. The labor adjustment cost model delivers considerable biases. For example, κ (investment adjustment cost), χ_p (price indexation), and ϕ_π and ϕ_Y (Taylor rule coefficients on inflation and output growth) are understated. ρ_g and $100\sigma_g$ (parameters characterizing the stochastic process of government spending shocks) are also imprecise. The results of the capital adjustment cost model and the labor adjustment cost model suggest that frictions that create distortions may not be limited to those I assumed in the baseline model.

6. Practical considerations

So far I have argued that in current DSGE models, distortions due to misspecification arising from ignoring seasonality could be potentially large. However, this claim is based on an experiment in a considerably restricted setting. In particular, I have assumed that an econometrician has complete knowledge about the structure of the economy and the mechanism generating seasonality. In practice, such knowledge is not fully available. A researcher who ignores seasonality could be even worse off if she introduces a grossly misspecified mechanism of seasonality (Sims, 1993; Hansen and Sargent, 1993). Thus, researchers face an important trade-off on whether to explicitly model seasonality or not. In this section I propose a simple procedure that helps researchers in making this decision, and I demonstrate how to use it.

A key component of the proposed procedure is to allow a coherent comparison between seasonal and aseasonal DSGE models. Let $\mathbf{y}_{t,q}$ be a vector of time-series data in time t at quarter q. Then the mapping of data from the model is,

$$\ln \mathbf{y}_{t,q} = \widehat{\mathbf{s}}_{t,q} + \ln \mathbf{s}_q, \quad (2)$$

for the seasonal model and

$$\ln \mathbf{y}_{t,q} = \widehat{\mathbf{s}}_t + \ln \mathbf{s} + \ln \mathbf{k}_q, \quad (3)$$

for the aseasonal model. Here $\mathbf{k}_q$ is a vector of seasonal dummies that is meant to capture seasonal variations in the data that cannot be explained by the aseasonal model. The orthogonal decomposition between $\widehat{\mathbf{s}}_t$ and $\mathbf{k}_q$ is consistent with a standard practice of seasonal adjustment. $\mathbf{k}_q$ is jointly estimated with structural parameters of the aseasonal model. Then a researcher can evaluate the fit across specifications by comparing the marginal likelihoods.[24]

I apply the mappings (2) and (3) to the simulated data used in the main experiment. The goal of the exercise is to demonstrate the usefulness of the approach for determining whether or not to explicitly model seasonality when there is potential danger of misspecification. I consider three examples of misspecification. The first example is the misspecification arising from ignoring seasonality (i.e., misspecification arising from using the aseasonal model), which has been the main focus of this paper. The second is the misspecification arising from the structure of the economy that is not directly related to the mechanism generating seasonality. In particular, I assume that a researcher thinks that the central bank responds to the output gap but not to output growth:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left\{ \left(\frac{\pi_t}{\pi_t^\star}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_q}\right)^{\phi_Y} \right\}^{1-\rho_R} e^{\epsilon_{R,t}},$$

$$\epsilon_{R,t} \sim N(0, \sigma_R^2).$$

In the third example, the mechanism generating seasonality is misspecified. I assume that a researcher thinks that the seasonality in preference originates from shifts in the habit term b_q, but not τ_q. To compare the biases across specifications in a systematic way, as in Ferroni (2011) I consider the following quadratic loss function that measures overall distortions:

$$QL = (\overline{\theta} - \theta_{true})\Sigma_\theta(\overline{\theta} - \theta_{true})',$$

where $\overline{\theta} = 1/N\sum_{i=1}^{N}\theta_i$ and $\Sigma_\theta = 1/N\sum_{l=1}^{L}(\overline{\theta}-\theta_i)(\overline{\theta}-\theta_i)'$. Thus a larger value of quadratic loss implies larger bias.[25]

Table 8 presents the results under various model specifications. Two things emerge. First, when seasonality is explicitly modeled, parameter biases are likely to be modest even when other parts of the model are misspecified. In contrast, when seasonality is not modeled, the biases are large. This suggests that misspecification arising from ignoring seasonality is practically important in potentially misspecified models. For example, when both the Taylor rule and the mechanism for seasonality is misspecified, the quadratic loss is 0.0209. This is less than half compared to when seasonality is not modeled (0.1238 and 0.0791).[26] Second, although a smaller marginal likelihood does not necessarily imply larger parameter biases, it appears to be a relatively good indicator for a measure of biases. Other forms of misspecifications not considered here may imply substantially larger biases. Nevertheless, a researcher can diagnose the presence of misspecification by comparing marginal likelihoods.

Table 8
Comparison across alternative model specifications.

Source of misspecification			Marginal likelihood	Quadratic loss
Seasonality not modeled	Misspecified Taylor rule	Misspecified seasonality		
			5824.4	0.0039
	✓		5802.3	0.0157
		✓	5788.4	0.0135
	✓	✓	5761.8	0.0209
✓			5259.2	0.1238
✓	✓		5226.4	0.0791

Notes: the marginal likelihoods are calculated based on the modified harmonic mean estimator by Geweke (1999). For the truncation value I use $p = 0.5$. Other values deliver similar results.

7. Conclusion

Conventional wisdom among economists is that seasonal adjustments represent an innocuous data filtering that allows econometricians to focus on the estimation of objects of interest with little distortion. In this paper, I have challenged that view. Using a state-of-the-art DSGE model that can match salient features of U.S. seasonal and nonseasonal fluctuations, I showed that estimation using seasonally adjusted data leads to important distortions. The problem cannot be mitigated by constructing alternative seasonal adjustment filters, as the distortions still arise in large sample environments with "ideal" filters. This is because the effects of seasonality, which are magnified by several frictions built into the model, are propagated across the entire frequency domain.

One limitation of the analysis in this paper is that I have focused my attention on a full-information likelihood approach. Since the main reason for the distortions is that agents have different responses to shocks across seasons, we may be able to obtain better estimates by using only moments that do not condition on a season. For example, as I mentioned in Section 4, since "seasonally adjusted" impulse responses from the seasonal model and impulse responses from the aseasonal model are almost identical when the same parameter values are used, it seems reasonable to perform indirect inference by matching impulse responses of seasonally adjusted data and the aseasonal model. A systematic investigation of this idea is left for future research.

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[24] In this respect, the proposed procedure resembles the one-step approach of trend estimation in Ferroni (2011).

[25] In the vectors θ and θ_{true}, I only include the structural parameters that are common between the seasonal and aseasonal models.

[26] When X-12-Arima-filtered data are used (Table 4), the quadratic loss is 0.0669.

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