Journal of Economic Dynamics & Control 78 (2017) 1–25

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

The uncertainty multiplier and business cycles

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ARTICLE INFO

Article history:
Received 18 November 2016
Revised 17 February 2017
Accepted 21 February 2017
Available online 2 March 2017

Keywords:
Uncertainty
Learning
Business cycles

ABSTRACT

I study a business cycle model where agents learn about the fundamentals by accumulating capital. During recessions, agents invest less, and this generates noisier estimates of macroeconomic conditions and an increase in uncertainty. The endogenous increase in aggregate uncertainty further reduces economic activity and thus gives rise to a multiplier effect that amplifies aggregate fluctuations. To discipline learning dynamics, I parametrize the model so that it matches not only standard business cycle moments but also survey data on macroeconomic forecasts. I find that the uncertainty multiplier amplifies output standard deviation by 16%.

1. Introduction

How important is time-varying aggregate uncertainty for macroeconomic fluctuations? A rapidly growing literature argues that shocks to uncertainty are a significant source of business cycle dynamics—see, for example, Bloom (2009), Fernández-Villaverde et al. (2011), Gourio (2012), and Christiano et al. (2014). However, the literature faces two important criticisms. In uncertainty shock theories, an exogenous increase in the volatility of structural shocks cause recessions. First, fluctuations in uncertainty may be, at least partially, endogenous.[2] The distinction is crucial because if uncertainty is an equilibrium object that is coming from agents' actions, policy experiments that treat uncertainty as exogenous are subject to the Lucas critique. Second, some authors (Bachmann and Bayer 2013, Born and Pfeifer 2014, and Chugh 2016) have argued that, given small and transient fluctuations in measured volatility, changes in uncertainty have negligible effects.

Motivated by these criticisms, I present a business cycle model where the level of economic activity influences the level of aggregate uncertainty through informational frictions. The endogenous movement in uncertainty, in turn, affects the level of economic activity through precautionary behavior and countercyclical markups. My goal is to quantify the role of this two-way feedback between uncertainty and economic activity in explaining the fluctuations of macroeconomic variables.

I embed the idea of asymmetric learning (Veldkamp 2005 and Van Nieuwerburgh and Veldkamp 2006) on investment into a standard DSGE framework with several real and nominal rigidities (Christiano et al. 2005). Since investment is the most volatile component of the business cycle and fluctuations in various measures of aggregate uncertainty (based on, for example, GDP forecasts or VIX) are also quite large, asymmetric learning through investment is a natural choice for modeling countercyclical uncertainty that depends on the level of economic activity.[3]

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[1] This work is based on the first chapter of my Ph.D. dissertation at the University of Pennsylvania. I am grateful to my advisor, Jesús Fernández-Villaverde, for his guidance and encouragement at every stage of this project. I also thank my dissertation committee, Dirk Krueger and Frank Schorfheide, the two anonymous referees, and seminar participants at numerous places whose comments led to a substantial improvement of the paper.

[2] See Bachmann et al. (2013) for supporting VAR evidence.

[3] Learning about aggregate productivity through production, for example, would generate much smaller fluctuations in uncertainty and hence amplification. In general, it is also possible to study asymmetric learning on firms' idiosyncratic demand conditions. I did not pursue this route because aggregate

http://dx.doi.org/10.1016/j.jedc.2017.02.008
0165-1889/

I introduce information frictions by subjecting the economy to aggregate shocks that agents cannot directly observe, namely, shocks to the marginal efficiency of investment and shocks to the depreciation rate of capital. Because the former are persistent while the latter are transitory, what matters for agents' decision is the evolution of the efficiency of investment. Agents use the path of capital stock and investment to form their estimates in a Bayesian manner.[4] However, the capital stock is not perfectly revealing about the unobservable shocks because it is subject to a non-invertibility problem: Agents cannot tell whether an unexpectedly high realization of capital stock is due to a high efficiency of investment or to a low depreciation rate of capital.

In the model, the level of investment endogenously determines the informativeness of the capital stock about the shocks to the efficiency of investment. When agents invest less, their estimates are imprecise because the level of capital stock is largely determined by the realization of the depreciation shock. Conversely, when they invest more, their estimates are accurate because the current capital mostly reflects shocks to the efficiency of investment. Thus, uncertainty about the efficiency of investment becomes endogenously countercyclical over the business cycle.

The countercyclical uncertainty gives rise to a novel multiplier effect that amplifies business cycles. Imagine that the economy is hit by a negative shock that lowers investment (for example, an exogenous tightening of monetary policy). Since agents learn less about the current period shock to the efficiency of investment, uncertainty increases. This, in turn, further reduces investment and other economic activity because of countercyclical markups and the reduction in households' spending due to their precautionary behavior. The opposite channel works when the economy is hit by a positive shock. I call this amplification mechanism the *uncertainty multiplier*.

To measure the size of the uncertainty multiplier, I perform numerical simulations. I estimate the parameters using the simulated method of moments (SMM) so that the model matches the business cycle properties of the postwar U.S. quarterly data. An interesting challenge I face is that the choice of the variance parameters has important effects on the strength of learning dynamics. More specifically, when the variance of the depreciation shock is very small compared to that of the shock to the efficiency of investment, the capital stock is almost perfectly revealing about the shock to the efficiency of investment. Conversely, when the depreciation shock is very large, the capital stock is uninformative and little learning takes place. In both cases, fluctuations in aggregate uncertainty are negligible. To ensure that agents face a realistic amount of information frictions, I use moments from the Survey of Professional Forecasters (SPF) in the SMM so that the model replicates the properties of macroeconomic forecasts.

I find that, under a full set of real and nominal rigidities, endogenous uncertainty amplifies the standard deviation of output by 16-20%. The mechanism also amplifies other real variables, such as investment and hours, by a similar amount. The size of the amplification is nontrivial due to three main features of the model. First, the uncertainty process is volatile and persistent because it is tied to the movement of investment. Second, as in Basu and Bundick (2016) and Fernández-Villaverde et al. (2015), nominal rigidities amplify the effects of time-varying uncertainty because of the countercyclical markups. Third, real rigidities also amplify the effects because they affect the inference about the size of uncertainty. The third point stands in sharp contrast to the case of exogenous uncertainty, where real rigidities reduce the overall effect of uncertainty.

Finally, I provide an external validation of the theory by conducting three tests. First, I show that the model can account for a sizable fraction of fluctuations in uncertainty. Second, I show that the model can replicate the VAR impulse responses. In particular, the model can account for the negative relationship between output and uncertainty and it also reproduces gradual responses of the two variables. This is because in the model uncertainty is inversely related to investment, which exhibits hump-shaped dynamics, and this uncertainty in turn induces gradual adjustments by households. Finally, I conduct Granger causality tests between output and uncertainty and find that the results from the model are in line with the data. For both data and the model, the hypothesis that output does not Granger-cause uncertainty are rejected at the 5% level. On the other hand, the hypothesis that uncertainty does not Granger-cause output cannot be rejected at the 10% level. Intuitively, in my model output has an explanatory power for the future level of uncertainty because uncertainty is driven by changes in economic activity. Since there is no exogenous impulse to uncertainty that moves economic activity, once we control for lagged output, lagged uncertainty is not useful in forecasting the level of output.

The rest of the paper is organized as follows. The next section describes the contributions of this paper with respect to the existing literature. Section 3 presents the model and Section 4 discusses its solution and parameterization. Section 5 presents the results and discusses in detail the mechanisms that determine the size of the amplification. Section 6 conducts empirical tests of my model and Section 7 concludes.

2. Connections to the literature

This paper is related to several strands of the literature. First, it is related to a growing literature on macroeconomic uncertainty shocks. For example, Basu and Bundick (2016) study the effects of volatility shocks to aggregate technology and preference in a standard New Keynesian model. Bianchi et al. (2017) consider Knightian uncertainty shocks to firms' financial

dynamics become intractable due to nonlinearities at the firm level. In a related paper, Ilut and Saijo (2016) are able to study a model where firms learn about their demand through production because of their assumption of Knightian uncertainty.

[4] In the model, all information necessary for optimal learning is contained in the path of capital stock and investment. While agents have access to other endogenous variables, including prices, those variables do not reveal additional information about the unobservable shocks.

conditions and characterize their impacts on asset prices. Other examples include Gourio (2012), Fernández-Villaverde et al. (2015), Born and Pfeifer (2014), and Ilut and Schneider (2014).[5] A distinct but related literature studies the effects of micro-level uncertainty shocks. A leading example is a paper by Bloom (2009), who shows that an exogenous increase in the volatility of firm-level productivity reduces output through a "wait-and-see" effect due to investment irreversibility. See also Arellano et al. (2016), Christiano et al. (2014), Chugh (2016), Gilchrist et al. (2014), and Schaal (2015). In this paper, I show that time-varying aggregate uncertainty could be an important amplification (rather than an impulse) mechanism of the business cycle. The difference is important for a structural analysis because now uncertainty is an equilibrium object that is policy-variant.[6]

Another important difference from the macro uncertainty shock literature is that, in my model, changes in uncertainty are not necessarily followed by changes in realized volatility. The previous literature has focused on theories where uncertainty shocks generate actual changes in volatility of macro variables.[7] Here there is also an interesting parallel with the idiosyncratic uncertainty shock literature. Bloom (2009), who calibrates the uncertainty process based on the VIX, and Christiano et al. (2014), who estimate cross-sectional risk shocks using macroeconomic data, find relatively large effects of uncertainty shocks while Chugh (2016), who measures cross-sectional risk fluctuations using microeconomic evidence, finds smaller effects. The tension between those studies suggests that changes in perceived idiosyncratic uncertainty play a quantitatively distinct role from shocks to realized micro-level volatility in driving business cycles. In a similar way, my paper shows that fluctuations in perceived *macroeconomic* uncertainty can account for a sizable share of aggregate fluctuations even when they are not reflected in volatility of macro variables.

Several papers attempt to account for the countercyclical firm-level volatility through conventional first-moment shocks. For example, in Bachmann and Moscarini (2012), recessions induce firms to price-experiment, which in turn raises the cross-sectional dispersion of price changes. See also Decker et al. (2015), Kehrig (2015), and Tian (2015). Their focus is on the explanation of the movement of ex-post volatility. I go one step further by highlighting the implications of ex-ante uncertainty. This is why I can show that uncertainty is not only a by-product of agents' response to first-moment shocks, but also an important factor that affects real allocations.

The main mechanism of this paper builds on a literature on asymmetric learning, for example, Veldkamp (2005), Van Nieuwerburgh and Veldkamp (2006), Ordoñez (2013), and Görtz and Tsoukalas (2013). They argue that the time-varying speed of learning about the macroeconomic conditions could explain the asymmetries in growth rates over the business cycle. When the economy passes the peak of a boom, agents are able to precisely detect the slowdown, leading to an abrupt crash. At the end of the recession, agents' estimates about the extent of recovery are noisy, slowing reactions and delaying booms. My contribution is to explore the direct effects of endogenous fluctuations in perceived uncertainty about aggregate fundamentals that shift the levels of macro variables. Recessions are deeper because high uncertainty leads precautionary households to cut consumption. Booms are stronger for the opposite reason. This channel has been overlooked in the previous literature.[8]

Because the basic framework of this paper is based on Van Nieuwerburgh and Veldkamp (2006), I elaborate on how my model differ from theirs. First, while their model is a parsimonious real business cycle model, my model features nominal rigidities. The advantage of introducing sticky prices is that, as we see below, it induces co-movement in response to an endogenous change in uncertainty. Second, in their model the underlying unobservable fundamental follows a discrete Markov process while in my model it takes continuous values. This allows me to conduct impulse response analysis as in the standard model. Third, in my model agents learn about the marginal efficiency of investment through investment while in their paper learning about productivity occurs through production. As mentioned above, because investment is the most volatile component of the business cycle, my approach is a natural starting point to study fluctuations in uncertainty due to economic activity. Adding learning through production would further increase the size of the uncertainty multiplier.

A recent work by Fajgelbaum et al. (2016) independently develops similar ideas. There are two key distinctions. First, in their paper the level of aggregate uncertainty is related to the number of firms investing (extensive margin), while in my paper the level of investment influences the level of uncertainty (intensive margin). As in Van Nieuwerburgh and Veldkamp (2006), this specification allows me to perform realistic quantitative analysis without losing tractability.[9] Second, in their paper time-varying uncertainty feeds back into the level of economic activity through irreversible investment, while in my paper uncertainty influences business cycles through countercyclical markups due to nominal rigidities. The

[5] See also Fernández-Villaverde et al. (2011), who show that volatility shocks to real interest rates generate sizable contractions in an otherwise standard small open economy model.

[6] Bachmann et al. (2013) and Ludvigson et al. (2016) provide empirical evidence suggesting that high uncertainty is at least partly due to an endogenous response to recessions.

[7] The exception is Ilut and Schneider (2014), who construct a business cycle model where Knightian uncertainty shocks affect ambiguity-averse agents' confidence in probability assessments about the productivity process.

[8] My model also features imperfect information about the growth rate of the marginal efficiency of investment. In this respect, the paper is also related to the literature that studies business cycle implications of learning about the trend of technology growth (for example, Edge et al. 2007 and Tambalotti 2003).

[9] The method I use to solve the model may be of independent interest. I first analytically derive the law of motion for posterior variance (uncertainty) and then use a non-linear approximation to solve for the equilibrium dynamics. Senga (2015) and Ilut and Saijo (2016), for example, use related solution methods.

advantage of my approach is that the markup channel generates conditional comovements among real variables in response to time-varying uncertainty (Basu and Bundick 2016).

3. The model

I embed a learning problem into the capital accumulation process of a standard DSGE framework (Christiano et al. 2005, Justiniano et al. 2010, and Smets and Wouters 2007). This framework is a natural laboratory for my quantitative investigation, since it has now become the foundation of applied research in both academic and government institutions.

In the first subsection, I describe the information frictions. In the second subsection, I present the standard part of the model.

3.1. Learning and countercyclical uncertainty

I divide this subsection into several parts. First, I describe the setup. Second, I express the learning process as a Kalman filtering problem. Third, I present a simple example that illustrates the key properties of the filtering problem. Finally, I rewrite the capital accumulation process from the perspective of the agents. This clarifies the impact of changes in uncertainty on the agents' decision making.

3.1.1. Setup

The law of motion for capital held by each household, k_t, is subject to two types of aggregate disturbances:

$$k_t = (1-\delta_t)k_{t-1} + \mu_t i_{t-1}, \tag{1}$$

where i_{t-1} is the investment undertaken by individual households. The depreciation shock, δ_t, follows

$$\delta_t = \delta - \epsilon_{\delta,t},$$

where $\epsilon_{\delta,\,t}$ is i.i.d. distributed from a normal distribution with mean zero and variance σ_δ^2. The investment shock, μ_t, determines the marginal efficiency of investment. I assume that μ_t follows the stochastic process

$$\begin{aligned}\mu_t &= g_{t-1} + (1-\rho_\mu)\mu + \rho_\mu\mu_{t-1} + \epsilon_{\mu,t},\\ g_t &= \rho_g g_{t-1} + \epsilon_{g,t},\end{aligned}$$

where $\epsilon_{\mu,\,t}$ and $\epsilon_{g,\,t}$ are i.i.d. distributed from a normal distribution with mean zero and variance σ_μ^2 and σ_g^2, respectively. The growth shock, g_t, controls the growth rate of μ_t.[10] As we see below, one of the key assumptions here is that while the shock to depreciation is transitory, the investment shock is persistent. First, the proxy for the depreciation shock shows less persistence than that of the investment shock. The first-order autocorrelation of realized annual depreciation rate, calculated from the consumption of fixed capital, is 0.37 while for the Baa credit spread (Moody's seasoned Baa corporate bond yield relative to yield on 10-year treasury), a common proxy for the investment shock in the literature, the first-order autocorrelation at annual frequency is 0.68. Second, I also consider an alternative setting where I relax the assumption of an i.i.d. depreciation shock and allow persistence:

$$\delta_t = (1-\rho_\delta)\delta + \rho_\delta\delta_{t-1} - \epsilon_{\delta,t}. \tag{2}$$

As I argue below, as long as the persistence of the depreciation shock is less than that of the investment shock, the main theoretical results of the model survive. Quantitatively, I find that the size of the uncertainty multiplier is similar to the one in the baseline specification.

Agents cannot directly observe the current or previous values of δ_t, μ_t, and g_t. This informational assumption gives rise to a non-invertibility problem: Agents cannot tell whether an unexpectedly high realization of capital stock is due to a high efficiency of investment or to a low depreciation rate of capital. As a result, they face a signal-extraction problem in forecasting the evolution of the shocks. Agents use all available information, including the path of capital stock, to form their estimates.

As in Veldkamp (2005) and Van Nieuwerburgh and Veldkamp (2006), I rule out active experimentation and assume that agents are passive learners. In other words, households do not internalize the effects of their own actions on learning.[11] As a result, in equilibrium all households choose the same action and hence observing the law of motion of aggregate capital stock, K_t, given by

$$K_t = (1-\delta_t)K_{t-1} + \mu_t I_{t-1}, \tag{3}$$

does not reveal additional information about the underlying shock. This is because the law of motion (3) contains the same information about the underlying shock as the law of motion of individual capital stock (1). The assumption of passive

[10] The growth shock is not necessary for the main qualitative results. However, as I show below the shock helps match some of the survey data moments.

[11] The assumption of passive learning is common in the macro learning literature. See, for example, Primiceri (2006).

learning is important. If instead agents are active learners and take into account the effect of their capital and investment choices on the evolution of beliefs, individual households can perfectly back out the realization of underlying shocks by slightly deviating from the aggregate law of motion (3). However, there are two modifications to the baseline setup that would allow the model to generate countercyclical uncertainty even in an active learning environment. First, it is possible that agents do not observe the actions of others and that aggregate capital stock is measured with error. Second, even if agents can completely observe the action of others, those observations are irrelevant if the unobservable shocks (δ_t, μ_t, and g_t) are idiosyncratic.[12] In what follows, for tractability and computational reasons, I assume passive learning and use the law of motion of aggregate capital (3) to discuss the Kalman filtering problem.[13]

A literal interpretation of the depreciation shock is that it represents an exogenous change in the physical depreciation rate of capital. However, as in Gertler and Karadi (2011) and Liu et al. (2011), a broader interpretation is possible. For example, it can represent an economic obsolescence of capital. Alternatively, reallocation of capital may be subject to temporary frictions and could show up as a change in the "quality" of aggregate capital.

The investment shock was originally proposed by Greenwood et al. (1988). In a medium-scale DSGE model similar to the one employed in this paper, Justiniano et al. (2010) found that the shock is the most important driver of the U.S. business cycle. An important question is whether the realization of investment shock is reflected in prices such as the relative price of investment goods to consumption goods. To answer this question, it is useful first to recognize that, as highlighted in Justiniano et al. (2011), the investment shock itself can be decomposed into two separate shocks. The first shock is an investment-specific technology (IST) shock that represents disturbances that affect the transformation of consumption goods into investment goods. The second shock is a disturbance to the marginal efficiency of investment (MEI) that affects the transformation of investment goods into installed capital. In equilibrium, the IST shock generates variations in the relative price of investment goods but the MEI shock does not. Estimation results from several papers indicate that the investment shock mostly reflects variations in the MEI and not the IST.[14] Intuitively, this is because the observed variation in the price of investment goods is too small to play a prominent role. Motivated by the evidence, I assume that the investment shock does not affect the relative price of investment. This assumption implies that agents cannot back out the underlying shocks by observing prices. In Appendix C, I extend the baseline model so that the production of consumption, investment, and capital goods are decentralized in different sectors as in Justiniano et al. (2011). The decentralization is useful because it allows me to explicitly introduce both IST and MEI shocks while preserving the information friction. I find that the results from this extended model are very similar to that of the baseline model.

In previous research, the investment shock is often viewed as a reduced-form disturbance to financial frictions. For example, as explained in detail in Justiniano et al. (2011), the investment shock plays a similar role to that of net worth in Carlstrom and Fuerst (1997)'s model. In the original Carlstrom and Fuerst (1997) model, only entrepreneurs have access to a technology that transforms consumption goods into investment goods and their idiosyncratic productivity is private information. They finance their production using their own net worth and external funds but the external funds are subject to agency frictions. Intuitively, the investment shock plays a similar role to that of net worth because net worth lowers agency cost and acts as a shifter of a capital supply curve. This particular interpretation of the investment shock becomes problematic if applied directly to my model because net worth and the transformation of consumption goods into installed investment are observable in the original Carlstrom and Fuerst (1997) model. One way to introduce procyclical learning in a way that is consistent with models with financial frictions would be to modify the setting so that (i) entrepreneurs combine the investment goods they produced with capital that are subject to idiosyncratic and aggregate depreciation shocks, (ii) agents cannot observe other entrepreneurs' decisions and (iii) proxies at the aggregate level for the external finance premium, such as credit spreads or aggregate net worth, are measured with error. These assumptions ensure that even if agents know their idiosyncratic conditions, they do not fully reveal aggregate conditions and hence aggregate uncertainty continues to be countercyclical.

As summarized in Fig. 1, the timing of events is as follows: At the end of period $t-1$, agents choose their investment level I_{t-1} given the current capital level K_{t-1} and their estimates about the unobservable state. Then, at the beginning of period t, unobservable shocks are realized. Finally, after observing the level of new capital K_t, agents update the estimates.

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[12] Ilut and Saijo (2016) study a business cycle model where firms face Knightian uncertainty about their own profitability and need to learn it through production. Interestingly, in their model the degree of amplification is larger under active learning than under passive learning.

[13] The discussion of this paragraph extends to an environment where firms, instead of households, accumulate capital. Potential complications arise when a Calvo-type price stickiness is introduced in this environment since firms' capital decisions become heterogeneous. There are two ways to introduce price stickiness without generating the heterogeneity. First, one can assume a Rotemberg-type quadratic price adjustment cost instead of the Calvo friction. Second, as in Bernanke et al. (1999), one can assume that the monopolistic competition and Calvo pricing happen at the "retail" level. With these assumptions, firm-level and aggregate law of motions of capital coincide under passive learning and hence the distinction becomes irrelevant when solving the filtering problem.

[14] For example, in Justiniano et al. (2010), they find that the implied investment shock from the estimation is four times as volatile and weakly correlated with the relative price of investment goods. They do not use the relative price of investment in the estimation. Schmitt-Grohe and Uribe (2012), on the other hand, use the investment price to identify all investment shocks and find that the shock can explain only a small fraction of business cycles. Finally, Justiniano et al. (2011) estimate a DSGE model that allows for both IST and MEI shocks and use the investment price to identify the IST shock. They find that MEI shock is the dominant driver of the business cycle.

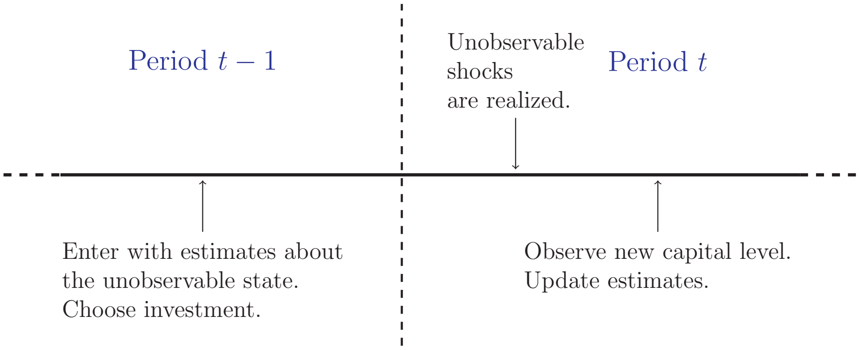

Fig. 1. Timing of events.

3.1.2. More on the setup: concrete examples

The information friction in the model is set up in a fairly stylized way. However, the same qualitative feature arises in a more concrete environment. Below, I provide two examples.

First, one can consider a setting where the aggregate capital stock is composed of distinct vintages that are hit by stochastic depreciations.[15] The process of μ_t is persistent, reflecting the view that the productivity of "close-by" vintages are similar. Let K_t denote the sum of all efficiency units of capital available for production in period t:

$$K_t = \sum_{s=0}^{S} K_{t,s}$$

where $K_{t,\,s}$ is capital of vintage s that is available at time t. The vintages of capital evolve according to

$$K_{t,s} = \begin{cases} \mu_t I_{t-1} & \text{if } s = 0 \\ (1-\delta)K_{t-1,s-1} + \epsilon^{\delta}_{t,s} & \text{if } s \geq 1 \end{cases}$$

where $\epsilon^{\delta}_{t,s}$ is a stochastic capital depreciation to vintage s at time t. $\epsilon^{\delta}_{t,s}, s = 1, \ldots, S$ are distributed from a multivariate normal distribution with mean zero and covariance matrix Σ_δ. While I assume that draws are independent across time, I still allow for contemporaneous correlations among $\epsilon^{\delta}_{t,s}$. The evolution of aggregate capital is then given by

$$K_t = (1-\delta)K_{t-1} + \tilde{\epsilon}^{\delta}_t + \mu_t I_{t-1}$$

where $\tilde{\epsilon}^{\delta}_t \equiv \sum_{s=1}^{S} \epsilon^{\delta}_{t,s}$ are distributed independently across time from a normal distribution with mean zero and some variance $\tilde{\sigma}_\delta$.[16] Thus, in this example capital depreciation is transitory while the marginal efficiency of investment is persistent.

Second, as will be discussed in detail below, an important feature of the capital accumulation technology is that it generates a procyclical signal-to-noise ratio. In Appendix D, I show how this feature also arises from aggregation of investment units with common and idiosyncratic shocks.

3.1.3. The Kalman filtering problem

Agents update their estimates about μ_t and g_t in an optimal (Bayesian) manner. The learning process can be expressed as a Kalman filtering problem:

$$\begin{bmatrix} \mu_t \\ g_t \end{bmatrix} = \begin{bmatrix} (1-\rho_\mu)\mu \\ 0 \end{bmatrix} + \begin{bmatrix} \rho_\mu & 1 \\ 0 & \rho_g \end{bmatrix} \begin{bmatrix} \mu_{t-1} \\ g_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{\mu,t} \\ \epsilon_{g,t} \end{bmatrix}, \tag{4}$$

$$K_t - (1-\delta)K_{t-1} = [I_{t-1} \quad 0] \begin{bmatrix} \mu_t \\ g_t \end{bmatrix} + K_{t-1}\epsilon_{\delta,t}. \tag{5}$$

Eq. (4) is the state equation that characterizes the evolution of the unobservable state. Eq. (5) is the measurement equation that describes the observables as a linear function of the underlying state. I point out two things regarding the measurement equation. First, $\epsilon_{\delta,\,t}$ serves as a measurement error in the filtering system. Second, unlike standard time-invariant systems, the coefficient matrices are time-varying.

The key property of the system is that the signal-to-noise ratio is procyclical, which follows from the fact that $\frac{I_{t-1}}{K_{t-1}}$ is procyclical. The flip side implication of this property is that *uncertainty is countercyclical*. Denote Σ_t as the error-covariance

[15] Görtz and Tsoukalas (2013) also consider a model with the vintage view of capital stock.

[16] The depreciation shock is additive in this example while it is attached to capital stock in the baseline model. The difference matters little because capital stock is acyclical.

matrix of the unobservable states,

$$\Sigma_t = \begin{bmatrix} \text{Var}_t(\mu_t - \tilde{\mu}_t) & \text{Cov}_t(\mu_t - \tilde{\mu}_t, g_t - \tilde{g}_t) \\ \cdots & \text{Var}_t(g_t - \tilde{g}_t) \end{bmatrix},$$

then the elements of Σ_t are decreasing in $\frac{I_{t-1}}{K_{t-1}}$. Intuitively, when agents invest more, their estimates about the efficiency of investment are accurate because current capital mostly reflects shocks to the efficiency of investment. Conversely, their estimates are imprecise when they invest less because the level of capital stock is largely determined by the realization of the depreciation shock. In the extreme case of zero investment, capital stock does not contain any information about the marginal efficiency of investment.

3.1.4. Understanding why uncertainty is countercyclical

I explain how a procyclical signal-to-noise ratio leads to countercyclical uncertainty by going through a simple example. In particular, assume that there is no growth shock.[17] Then the filtering problem reduces to

$$\mu_t = (1-\rho_\mu)\mu + \rho_\mu \mu_{t-1} + \epsilon_{\mu,t}, \tag{6}$$

$$y_t = I_{t-1}\mu_t + K_{t-1}\epsilon_{\delta,t}, \tag{7}$$

where (6) is the state equation and (7) is the measurement equation. I define $y_t \equiv K_t - (1-\delta)K_{t-1}$. In period $t-1$, agents enter with the mean estimate $\tilde{\mu}_{t-1}$ and its associated error variance $\Sigma_{t-1} \equiv \text{Var}_{t-1}(\mu_{t-1} - \tilde{\mu}_{t-1})$. Then, the period $t-1$ prediction of μ_t and its associated error variance is given by

$$\tilde{\mu}_{t|t-1} = (1-\rho_\mu)\mu + \rho_\mu \tilde{\mu}_{t-1}$$
$$\Sigma_{t|t-1} = \rho_\mu^2 \Sigma_{t-1} + \sigma_\mu^2$$

After observing the outcome y_t, they update their estimates according to

$$\tilde{\mu}_t = \tilde{\mu}_{t|t-1} + Gain_t(y_t - I_{t-1}\tilde{\mu}_{t|t-1}),$$

where $Gain_t$ is the Kalman gain and is given by

$$Gain_t = \underbrace{\frac{I_{t-1}^2 \Sigma_{t|t-1}}{I_{t-1}^2 \Sigma_{t|t-1} + K_{t-1}^2 \sigma_\delta^2}}_{\text{Informativeness of observation}} \cdot \underbrace{\frac{1}{I_{t-1}}}_{\text{Adjustment}}.$$

The first term represents the informativeness of observation y_t and is given by the variance of the signal divided by the total variance (the variance of the signal and noise). The term is increasing in $\frac{I_{t-1}}{K_{t-1}}$. The second term is the scale adjustment term reflecting the fact that μ_t is multiplied by I_{t-1} in the observation.

The error variance associated with $\tilde{\mu}_t$ is given by

$$\begin{aligned} \Sigma_t &= (1 - Gain_t I_{t-1})\Sigma_{t|t-1} \\ &= \underbrace{\frac{K_{t-1}^2 \sigma_\delta^2}{I_{t-1}^2 \Sigma_{t|t-1} + K_{t-1}^2 \sigma_\delta^2}}_{\text{Un-informativeness of observation}} \cdot \Sigma_{t|t-1}. \end{aligned}$$

The first line says that the error shrinks as we learn more from the observation; the error is decreasing in the size of the Kalman gain. The second line says that the error variance is increasing in the un-informativeness of the observation (the variance of noise divided by the total variance). Since the un-informativeness term is decreasing in $\frac{I_{t-1}}{K_{t-1}}$, Σ_t is decreasing in $\frac{I_{t-1}}{K_{t-1}}$. Since investment is much more volatile than capital, $\frac{I_{t-1}}{K_{t-1}}$ moves almost proportionally to I_{t-1}. Thus, less investment leads to more uncertainty.

3.1.5. Implications of time-varying uncertainty from the perspective of the agents

How do changes in uncertainty about the current efficiency of investment affect agents' decision making? The key insight here is that, because shocks to the efficiency of investment are persistent, uncertainty about the current state translates into uncertainty about the future realization of capital.

To see this, it is useful to rewrite the capital accumulation equation from the perspective of the agent at period $t-1$:

$$K_t = (1-\delta_t)K_{t-1} + (\tilde{\mu}_{t|t-1} + u_t)I_{t-1},$$

where $\tilde{\mu}_{t|t-1}$ is the mean forecast of μ_t at time $t-1$ and u_t is normally distributed with mean zero and variance $\sigma_{u,t}^2$. The innovation u_t takes into account not only the exogenous innovation to μ_t, but also its estimation error:

[17] In the Appendix, I provide a full derivation with the growth shock.

$$\begin{aligned} u_t &= \mu_t - \tilde{\mu}_{t|t-1} \\ &= (g_{t-1} - \tilde{g}_{t-1}) + \rho_\mu(\mu_{t-1} - \tilde{\mu}_{t-1}) + \epsilon_{\mu,t}, \end{aligned}$$

and hence its variance is given by

$$\sigma^2_{u,t} = \rho^2_\mu \Sigma^{11}_{t-1} + 2\rho_\mu \Sigma^{12}_{t-1} + \Sigma^{22}_{t-1} + \sigma^2_\mu.$$

Thus, the fluctuation in uncertainty shows up as a fluctuation in the variance of the innovation to the mean forecast of the marginal efficiency of investment. Moreover, this fluctuation in variance is persistent to the extent that investment is persistent.

This intuition carries over to the case of unobserved persistent depreciation shock. In this case, the capital accumulation equation can be rewritten as

$$K_t = \{1 - (\delta_{t|t-1} + e_t)\}K_{t-1} + (\tilde{\mu}_{t|t-1} + u_t)I_{t-1},$$

where $\delta_{t|t-1}$ is the mean forecast of δ_t at time $t-1$ and the innovation e_t is given by

$$\begin{aligned} e_t &= \delta_t - \tilde{\delta}_{t|t-1} \\ &= \rho_\delta(\delta_{t-1} - \tilde{\delta}_{t-1}) - \epsilon_{\delta,t}. \end{aligned}$$

Since the effect of an estimation error about δ_t to the innovation e_t is discounted by ρ_δ, as long as the persistence of the depreciation shock is sufficiently lower than that of the investment shock, the overall uncertainty (the sum of the variances of e_t and u_t) is countercyclical. Intuitively, during recessions estimates about the marginal efficiency of investment becomes noisier but estimates of the depreciation shock becomes more precise. The increase in the variance of u_t is larger than the reduction of the variance of e_t as long as ρ_δ is sufficiently lower than ρ_μ.

3.2. Standard part of the model

I now describe other components of the model. The economy is composed of a final-goods sector, intermediate-goods sector, household sector, employment sector, and a central bank. I start by describing the production side of the economy.

3.2.1. The final-goods sector

In each period t, the final goods, Y_t, are produced by a perfectly competitive representative firm that combines a continuum of intermediate goods, indexed by $j \in [0, 1]$, with technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\theta_p - 1}{\theta_p}} dj\right]^{\frac{\theta_p}{\theta_p - 1}}.$$

$Y_{j,\,t}$ denotes the time t input of intermediate good j and θ_p controls the price elasticity of demand for each intermediate good. The demand function for good j is

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_p} Y_t, \tag{3.1.4}$$

where P_t and $P_{j,\,t}$ denote the price of the final good and intermediate good j, respectively. Finally, P_t is related to $P_{j,\,t}$ via the relationship

$$P_t = \left[\int_0^1 P_{j,t}^{1-\theta_p} dj\right]^{\frac{1}{1-\theta_p}}.$$

3.2.2. The intermediate-goods sector

The intermediate-goods sector is monopolistically competitive. In period t, each firm j rents $K_{j,\,t}$ units of capital stock from the household sector and buys $H_{j,\,t}$ units of aggregate labor input from the employment sector to produce intermediate good j using technology

$$Y_{j,t} = z_t K^\alpha_{j,t} H^{1-\alpha}_{j,t}.$$

z_t is the level of total factor productivity that follows

$$z_t = (1 - \rho_z)z + \rho_z z_{t-1} + \epsilon_{z,t},$$

where $\epsilon_{z,\,t}$ is i.i.d. distributed from a normal distribution with mean zero and variance σ^2_z.[18]

Firms face a Calvo-type price-setting friction: In each period t, a firm can reoptimize its intermediate-goods price with probability $(1 - \xi_p)$. Firms that cannot reoptimize index their price according to the steady-state inflation rate, π.

[18] I specify the productivity process in levels rather than in logs so that it is consistent with the process of the marginal efficiency of investment. During the simulation exercise, the level of productivity never falls below zero. The same remark applies to the preference shock introduced below.

3.2.3. The household sector

There is a continuum of households, indexed by $i \in [0, 1]$. In each period, household i chooses consumption C_t, investment I_t, bond purchases B_t, and nominal wage $W_{i,t}$ to maximize utility:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s d_{t+s} \left[\frac{(C_{t+s} - bC_{t+s-1})^{1-\sigma}}{1-\sigma} - \frac{H_{i,t+s}^{1+\eta}}{1+\eta} \right],$$

where β is a discount factor, b represents consumption habit, σ controls the degree of risk aversion, η controls (the inverse of) the Frisch labor supply elasticity, and $H_{i,t}$ is the number of hours worked. d_t is a preference shock that follows

$$d_t = (1-\rho_d)d + \rho_d d_{t-1} + \epsilon_{d,t},$$

where $\epsilon_{d,t}$ is i.i.d. distributed from a normal distribution with mean zero and variance σ_d^2.

The household's budget constraint is

$$P_t C_t + P_t I_t + B_t \le W_{i,t} H_{i,t} + R_t^k K_t + R_{t-1} B_{t-1} + D_t + A_{i,t},$$

where R_t^k is the rental rate of capital, K_t is the stock of capital,[19] R_{t-1} is the gross nominal interest rate from period $t-1$ to t, and D_t is the combined profit of all the intermediate-goods firms distributed equally to each household. I assume that households buy securities, whose payoffs are contingent on whether they can reoptimize their wage.[20] $A_{i,t}$ denotes the net cash inflow from participating in state-contingent security markets at time t.

As in Christiano et al. (2005), I add an investment adjustment cost to the capital accumulation equation described above:

$$K_t = (1-\delta_t)K_{t-1} + \mu_t \left(1 - S\left(\frac{I_{t-1}}{I_{t-2}}\right)\right) I_{t-1}, \tag{8}$$

where

$$S\left(\frac{I_{t-1}}{I_{t-2}}\right) = \frac{\kappa}{2}\left(\frac{I_{t-1}}{I_{t-2}} - 1\right)^2,$$

with $\kappa > 0$.[21] Other components of the capital accumulation, like the stochastic process of shocks or the informational structure, are exactly the same as described in the previous section.

3.2.4. The employment sector and wage setting

In each period t, a perfectly competitive representative employment agency hires labor from households to produce an aggregate labor service, H_t, using technology

$$H_t = \left[\int_0^1 H_{i,t}^{\frac{\theta_w - 1}{\theta_w}} di \right]^{\frac{\theta_w}{\theta_w - 1}},$$

where $H_{i,t}$ denotes the time t input of labor service from household i and θ_w controls the price elasticity of demand for each household's labor service. The agency sells the aggregated labor input to the intermediate firms for a nominal price of W_t per unit. The demand function for the labor service of household i is

$$H_{i,t} = \left(\frac{W_{i,t}}{W_t}\right)^{-\theta_w} H_t,$$

where $W_{i,t}$ denotes the nominal wage rate of the labor service of household i. W_t is related to $W_{i,t}$ via the relationship

$$W_t = \left[\int_0^1 W_{i,t}^{1-\theta_w} di \right]^{\frac{1}{1-\theta_w}}.$$

Households face a Calvo-type wage-setting friction: In each period t, a household can reoptimize its nominal wage with probability $(1-\xi_w)$. Households that cannot reoptimize index their wage according to the steady-state inflation rate, π.

[19] Note that K_t is the capital stock after the period t shock to the capital accumulation equation is realized.

[20] The existence of state-contingent securities ensures that households are homogeneous with respect to consumption and asset holdings, even though they are heterogeneous with respect to the wage rate and hours because of the idiosyncratic nature of the timing of wage reoptimization. See Christiano et al. (2005).

[21] With the investment adjustment cost, the measurement Eq. (5) is rewritten as

$$K_t - (1-\delta)K_{t-1} = \left[\left(1 - S\left(\frac{I_{t-1}}{I_{t-2}}\right)\right) I_{t-1} \quad 0 \right] \begin{bmatrix} \mu_t \\ g_t \end{bmatrix} + K_{t-1}\epsilon_{\delta,t}. \tag{9}$$

Note that since the adjustment cost is pre-determined, we can still continue to use the Kalman filter.

3.2.5. The central bank, resource constraint, and equilibrium

The central bank follows a Taylor rule with interest-rate smoothing:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left\{ \left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_Y} \right\}^{1-\rho_R} \exp(\epsilon_{R,t}),$$

where R is the steady-state level of the nominal interest rate, ρ_R is the persistence of the rule, and ϕ_π and ϕ_Y are the size of the policy response to the deviation of inflation and output growth from their steady states, respectively. $\epsilon_{R,\,t}$ is a monetary policy shock and is i.i.d. distributed from a normal distribution with mean zero and variance σ_R^2.

Finally, the aggregate resource constraint is $C_t + I_t = Y_t$. I employ a standard sequential market equilibrium concept and hence its formal definition is omitted.

4. Model solution and parameterization

I follow Fernández-Villaverde et al. (2011) and solve the model using a third-order perturbation method around its deterministic steady state.[22] I use perturbation because the model has many state variables and it is the only method that delivers an accurate solution in a reasonable amount of time (Aruoba et al. 2006). The third-order approximation is necessary because my purpose is to analyze the direct impact of endogenous changes in perceived uncertainty about the efficiency of investment. In a standard first-order approximation, changes in uncertainty play no role since the decision rules of agents are forced to follow a certainty equivalence principle. In the second-order approximation, changes in uncertainty only appear in the decision rules as cross-product terms with other state variables. Only in the third-order approximation do changes in uncertainty show up as an independent term.

The parameterization of the model is done in two steps. In the first step, I fix several parameter values following micro evidence or estimates found in other papers. In the second step, I estimate the remaining parameters using the simulated method of moments (SMM) to match data moments. The first step reduces the number of parameters to be estimated and thus sharpens the exercise in the second step.

The discount factor, β, is set so that the model steady-state interest rate implied by the Euler equation matches that of the data. The capital share is set to 0.3. $\delta = 0.02$ implies an annual depreciation rate of 8%. The elasticity of goods demand $\theta_p = 21$ and labor demand $\theta_w = 21$ are consistent with previous estimates, for example, Altig et al. (2011).

I set $\sigma = 2$ and the habit persistence parameter is set to $b = 0.75$. The value of σ is close to the estimated value in Smets and Wouters (2007) and used in, for example, Fernández-Villaverde et al. (2015). The degree of habit persistence is in line with the estimates found in Smets and Wouters (2007) and Justiniano et al. (2010). As emphasized in Boldrin et al. (2001), a strong habit persistence parameter helps to account for various asset pricing puzzles. Chetty et al. (2011) suggest a Frisch elasticity of labor supply of 0.5 for a macro model that does not distinguish between intensive and extensive margins. This leads to $\eta = 2$.

The Calvo price and wage parameters imply an average duration of one year. As found in Smets and Wouters (2007) and Justiniano et al. (2010), prices and wages need to be sufficiently sticky in order to account for the inflation and wage dynamics in the data. Turning to the monetary policy parameter, I match the steady-state inflation rate to its historical mean. The Taylor rule coefficients feature inertia with a strong response to inflation and a weak response to output growth (Levin et al. 2006, Smets and Wouters 2007, and Justiniano et al. 2010). The persistence coefficients for the technology shock and preference shock are set to $\rho_z = 0.95$ (Cooley and Prescott 1995) and $\rho_d = 0.22$ (Smets and Wouters 2007), respectively.

To determine the values of other parameters, I choose them so that the moments simulated from the model matches the selected moments in the data.[23] Specifically, let $\theta = \{\kappa, \rho_\mu, \rho_g, \sigma_z, \sigma_d, \sigma_R, \sigma_\mu, \sigma_g, \sigma_\delta\}$ denote a vector that collects 9 parameters to be estimated. ψ_0 is a vector that collects 9 data moments which not only include standard macro moments but also moments from the survey data:

- Macroeconomic variables:
  Standard deviations of output, investment, and consumption.
  Correlations of investment with respect to output.
  Autocorrelation of output, investment, and consumption.

[22] The computation is carried out with Dynare (http://www.dynare.org/).

[23] To simulate the model, I use the pruning procedure as described in Kim et al. (2008) and Den Haan and De Wind (2012). I compute a total of 200 replications of 272 period simulations. I throw away the initial 100 periods, which leaves me with the sample size of the US data (172 periods). For each sample I compute the business cycle moments and then take medians across 200 replications. I checked that the results are not driven by explosive behavior.

Table 1
Parameters and targets.

	Description	Value	Comments/Targets
Technology and preference			
β	Discount factor	0.9948	Historical mean of interest rate
θ_p	Goods demand elasticity	21	5% price markup (Altig et al. 2011)
θ_w	Labor demand elasticity	21	5% wage markup (Altig et al. 2011)
α	Capital share	0.3	30% capital share of income
δ	Depreciation rate	0.02	8% annual depreciation
σ	Preference parameter	2	Smets and Wouters (2007)
η	Inverse Frisch elasticity	2	Frisch elasticity = 0.5 (Chetty et al. 2011)
b	Habit persistence	0.75	Smets and Wouters (2007)
κ	Investment adj. cost	0.37	*Estimated using SMM*
ξ_p	Calvo price	0.75	Duration of price 4 quarters
ξ_w	Calvo wage	0.75	Duration of wage 4 quarters
Monetary policy			
π	SS inflation rate	1.0095	Historical mean of inflation rate
ρ_R	Taylor rule smoothing	0.9	Smets and Wouters (2007)
ϕ_π	Taylor rule inflation	2	Smets and Wouters (2007)
ϕ_Y	Taylor rule output growth	0.1	Smets and Wouters (2007)
Shock process			
ρ_z	Technology	0.95	Cooley and Prescott (1995)
ρ_d	Preference	0.22	Smets and Wouters (2007)
ρ_μ	Investment level	0.91	*Estimated using SMM*
ρ_g	Investment growth	0.85	*Estimated using SMM*
$100\sigma_z$	Technology	0.20	*Estimated using SMM*
$100\sigma_d$	Preference	4.2	*Estimated using SMM*
$100\sigma_R$	Monetary policy	0.006	*Estimated using SMM*
$100\sigma_\mu$	Investment level	0.25	*Estimated using SMM*
$100\sigma_g$	Investment growth	0.72	*Estimated using SMM*
$100\sigma_\delta$	Depreciation	0.014	*Estimated using SMM*

- Forecast errors from the Survey of Professional Forecasters:
  1st-order autocorrelation and mean size of forecast errors on nominal GDP growth.

The parameters θ are jointly selected to minimize the quadratic loss function:

$$\mathcal{L} = (\psi(\theta) - \psi_0)'(\psi(\theta) - \psi_0),$$

where $\psi(\theta)$ is a vector that collects simulated moments from the model. Table 1 summarizes the estimated parameter values along with other pre-determined parameters.

The identification of the standard deviation of the depreciation shock σ_δ needs further discussion. The parameter is important because it determines the strength of information frictions. When σ_δ is very small, the learning problem becomes trivial. When σ_δ is very large, agents learn little about the aggregate state. Thus in both cases, changes in the level of investment have a negligible effect on the level of uncertainty. I include data moments on forecast errors in the Survey of Professional Forecasters data in the SMM estimation to pin down σ_δ.[24] The first row in Table 2 reports statistical properties of the one-quarter-ahead median forecast errors on nominal GDP growth rate.[25] The second column shows that the forecast errors of GDP growth are positively autocorrelated. The third column shows the mean size of forecast errors (i.e., forecast precision). To illustrate how survey data moments facilitate identification, I report the model predictions of the forecast errors for various values of σ_δ.[26] First, note that for all values of σ_δ reported, the autocorrelations are positive. This is due to the relatively high persistence parameter of the investment growth shock, ρ_g. The forecast errors are autocorrelated because agents only gradually realize the change in growth rate in response to an innovation to g_t. As σ_δ increases, the autocorrelation in general decreases because of the additional noise in the filtering problem.[27] On the other hand, the size of the error increases with σ_δ simply because the information friction becomes more severe. While it is important to note that σ_δ is jointly estimated with other parameters and hence standard macro moments in the SMM also influence the estimation of σ_δ, Table 2 highlights the advantage of incorporating survey data moments in inferring the degree of information frictions.

[24] A similar calibration/estimation strategy has been used in, for example, Eusepi and Preston (2011) and Görtz and Tsoukalas (2013).

[25] I choose the nominal GDP growth rate because this is the longest forecast series available from the survey. Also, the forecasts do not appear to be biased because the time-series average of the forecast errors is very close to zero.

[26] For the computation of the numbers reported in this table, I only change the value of σ_δ and fix other parameters at the benchmark parameterize reported in Table 1.

[27] When σ_δ is small, the autocorrelation actually increases as σ_δ increases. For example, as shown in Table 2, the autocorrelation slightly increases from 0.17 to 0.18 when σ_δ increases from $100\sigma_\delta = 0$ to $100\sigma_\delta = 0.015$. This is because the effect of information friction, which causes agents to gradually learn about the change in the level of μ_t, offset the effect of an increase of the i.i.d. shock.

Table 2
Properties of forecast errors and output amplification for different values of σ_δ.

	$Corr(FE_t^{1Q}, FE_{t-1}^{1Q})$	$Mean(\|FE_t^{1Q}\|)$	Output amplification
Data	0.17	0.55	
Model			
$100\sigma_\delta = 0.000$	0.17	0.47	1.00
$100\sigma_\delta = 0.005$	0.18	0.48	1.11
$100\sigma_\delta = 0.014$	0.17	0.55	1.16
$100\sigma_\delta = 0.025$	0.14	0.67	1.23
$100\sigma_\delta = 0.050$	0.13	0.89	1.15

Notes: The forecast errors are multiplied by 100 to express them in percentage terms. The data statistics are calculated using the final data vintage. As a robustness check, I calculated the statistics using alternative data vintages and found that they are similar. For example, $(Corr(FE_t^{1Q}, FE_{t-1}^{1Q}), Mean(|FE_t^{1Q}|))$ for the first, the third, and the fifth vintages are (0.23, 0.47), (0.18, 0.52), and (0.20, 0.54), respectively.

Table 3
Business cycle moments.

	Std.	$Corr(Y_t, X_t)$	AR(1)
Data			
Output	1.61*	1.00	0.87*
Investment	6.31*	0.94*	0.87*
Consumption	0.93*	0.84	0.87*
Hours	1.99	0.88	0.92
Real wage	0.84	0.07	0.76
Inflation	0.29	0.18	0.48
Interest rate	0.41	0.34	0.75
Model			
Output	1.61*	1.00	0.89*
Investment	6.32*	0.90*	0.92*
Consumption	0.91*	0.59	0.81*
Hours	2.26	0.98	0.88
Real wage	1.07	-0.40	0.89
Inflation	0.46	0.09	0.68
Interest rate	0.26	-0.32	0.93

Notes: Both data and model moments are in logs, HP-filtered ($\lambda = 1600$), and multiplied by 100 to express them in percentage terms. Moments with asterisks denote those used in the SMM estimation.

As a preliminary diagnosis of the model's performance, I compare the business cycle moments from the data and the model in Table 3. The model matches the data reasonably well, even for moments that are not explicitly targeted in the estimation.

For robustness, I perform two additional exercises. First, as in Van Nieuwerburgh and Veldkamp (2006), my model generates asymmetric business cycles. When the economy passes the peak of a boom, agents are able to precisely detect the slowdown, leading to an abrupt crash. At the end of the recession, agents' estimates about the extent of recovery are noisy, slowing reactions and delaying booms. Under the baseline parameterization, however, the model underpredicts the asymmetry in the data. For example, the skewnesses of output and investment in the model are −0.07 and −0.07, respectively, while in the data they are −0.38 and −0.83, respectively. Thus, it is interesting to see how the results change when the model is parameterized in order to match the data closer in terms of business cycle asymmetry. When I present the size of business cycle amplification below, I also present the size of amplification when the skewnesses of output and investment, instead of the Survey of Professional Forecasters data, are used in the SMM estimation. Second, I consider a persistent depreciation shock as in (2). In this specification, I have a new parameter ρ_δ. To facilitate comparison with the baseline specification of an i.i.d. depreciation shock, I jointly estimate ρ_δ and rest of the 9 parameters using the 9 moments used in the original estimation.

5. Results

In this section, I present the results. First, by comparing impulse responses and business cycle moments, I quantify the size of the uncertainty multiplier. Then, I extensively discuss why countercyclical uncertainty leads to an amplification of business cycles and examine how the size of amplification depend on the structure of the economy. Finally, I examine the sensitivity of the size of the multiplier to different parameter values for the shock processes.

Table 4
The size of the amplification.

	Amplification $\sigma_{\text{With multiplier}}/\sigma_{\text{Without multiplier}}$		
	Baseline parameterization	Alternative parameterization	Persistent depreciation
Output	1.16	1.20	1.16
Investment	1.25	1.29	1.20
Consumption	1.05	1.05	1.05
Hours	1.24	1.24	1.18
Real wage	1.10	1.10	1.14
Inflation	1.03	1.03	1.06
Nominal interest	1.04	1.05	1.04

Notes: Both data and model moments are in logs and HP-filtered ($\lambda = 1600$). "Baseline parametrization" refers to results based on the SMM using survey data, "Alternative parametrization" refers to results based on the SMM using the skewnesses of output and investment, and "Persistent depreciation" refers to the model with a persistent depreciation shock.

5.1. The size of the uncertainty multiplier

I divide the presentation of the main results into two parts. First, I use impulse responses to explain the basic mechanism of the uncertainty multiplier. Second, I compute business cycle moments and measure the size of the multiplier.

5.1.1. Impulse response analysis

Before examining the impulse responses, I need to consider how to measure the effects of endogenous changes in perceived uncertainty about the efficiency of investment. One potential way is to compare the baseline model with a version of the model without any information friction (i.e., agents know the true value of the shocks). However, this approach is problematic since it confounds the effects of changes in the *variance* of the agents' estimates (which is the main focus of the paper) with the effects of changes in the *mean* of the estimates. Therefore, I consider a version of the model where agents' perception of the variance of the estimates is held constant but they still face information frictions. This way, I can precisely quantify the contribution of fluctuations in uncertainty to business cycle dynamics.

Recall that from the perspective of the agent at the end of period $t-1$, the capital accumulation equation can be rewritten as follows:

$$K_t = (1-\delta_t)K_{t-1} + (\tilde{\mu}_{t|t-1} + u_t)I_{t-1}, \tag{10}$$

where u_t is normally distributed with mean zero and variance $\sigma^2_{u,t}$. In the baseline model featuring the uncertainty multiplier, $\sigma^2_{u,t}$ is given by

$$\sigma^2_{u,t} = \rho^2_\mu \Sigma^{11}_{t-1} + 2\rho_\mu \Sigma^{12}_{t-1} + \Sigma^{22}_{t-1} + \sigma^2_\mu.$$

I shut down the uncertainty multiplier by fixing expectations over $\sigma^2_{u,t}$ at its steady-state level:

$$\sigma^2_{u,t} = \rho^2_\mu \Sigma^{11}_{ss} + 2\rho_\mu \Sigma^{12}_{ss} + \Sigma^{22}_{ss} + \sigma^2_\mu,$$

where Σ^{11}_{ss}, Σ^{12}_{ss}, and Σ^{22}_{ss} are the steady-state levels of Σ^{11}_t, Σ^{12}_t, and Σ^{22}_t. Intuitively, agents act as if ex-ante uncertainty is constant, even though the size of the ex-post forecast error about the marginal efficiency of investment is time-varying.

I examine the impulse responses to a one-standard-deviation technology shock and a monetary shock.[28] Fig. 2 shows that the output increase in response to a positive technology is stronger when the uncertainty multiplier is present (blue solid line) than without (black dashed line). This is because in the baseline model agents perceive a decline in uncertainty about the future realization of effective capital (increase in $\sigma_{u,t}$). The decline in uncertainty is due to an increase in investment originated from a positive technology shock. This decline in uncertainty contributes to the additional increase in output and other real variables compared to the case where the uncertainty multiplier is turned off ($\sigma_{u,t}$ is held constant). For nominal variables like inflation and the interest rate, the amplification is negligible. Fig. 3 similarly shows that the uncertainty multiplier magnifies the effect of a monetary policy shock on economic activities.

5.1.2. Business cycle moments

I measure the size of the uncertainty multiplier by computing the business cycle moments with and without the multiplier. Table 4 compares the standard deviations of output and other variables. First, consider the benchmark parameterization (labeled "Baseline parameterization"). The sizes of the amplification are nontrivial. In particular, the standard deviation of

[28] Impulse responses to other shocks are available upon request.

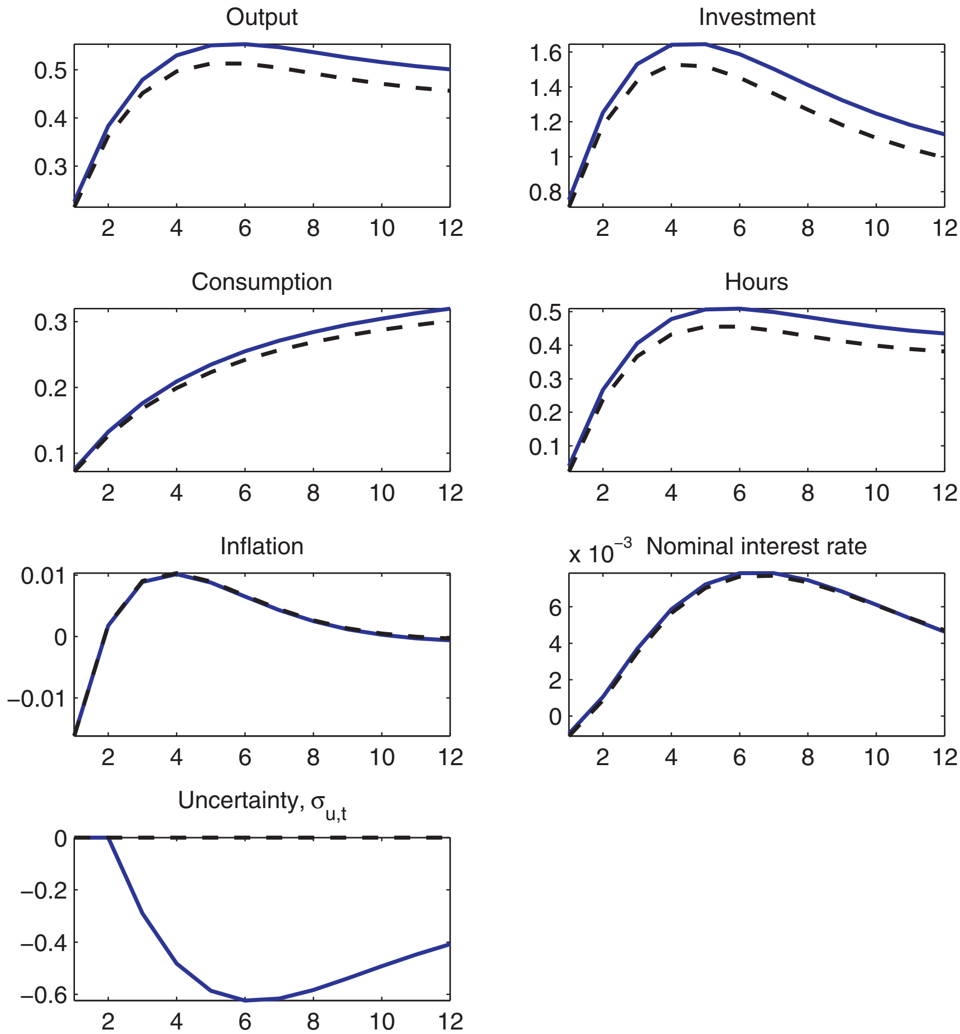

Fig. 2. Impulse response to a technology shock.
Notes: Blue solid lines are the baseline and black dashed lines are when the uncertainty multiplier is turned off. Since third-order approximations move the ergodic distribution of endogenous variables away from the steady state (Fernández-Villaverde et al. 2011), I report the impulse responses in terms of percent deviation from the ergodic mean. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

output is 1.16 times larger with the multiplier.[29] The uncertainty multiplier magnifies other real variables like investment and hours by a similar amount.[30] Consistent with the findings from the impulse response analysis, for inflation and the nominal interest rate the amplification is negligible. Next, consider the sizes of amplification when the skewnesses of output and investment are used in the SMM estimation (labeled "Alternative parameterization"). The sizes of amplification are larger than the baseline case. For example, the output amplification is 20% while it is 16% in the baseline parameterization. Intuitively, the amplification is larger because the business cycle asymmetry and hence fluctuations in uncertainty are larger when skewnesses are used in the SMM.[31] Finally, consider the model with a persistent depreciation shock (labeled

[29] The baseline numbers are derived from the HP-filtered ($\lambda = 1600$) moments. The multiplier is of similar magnitude when other detrending methods are used. For example, when I use linearly detrended moments, the uncertainty multiplier for output is 1.27.

[30] The amplification of consumption is smaller than other real variables because I used a flexible method (HP filter) to detrend the data. When I use linearly detrended moments, the uncertainty multipliers for investment, consumption, and hours are 1.27, 1.26, and 1.28, respectively.

[31] The model, however, still underpredicts the asymmetry in the data. The skewnesses of output and investment in the model are −0.10 and −0.23, respectively, while in the data they are −0.38 and −0.83, respectively. Other business cycle moments generated by the model are similar to those under the baseline parameterization and are available upon request.

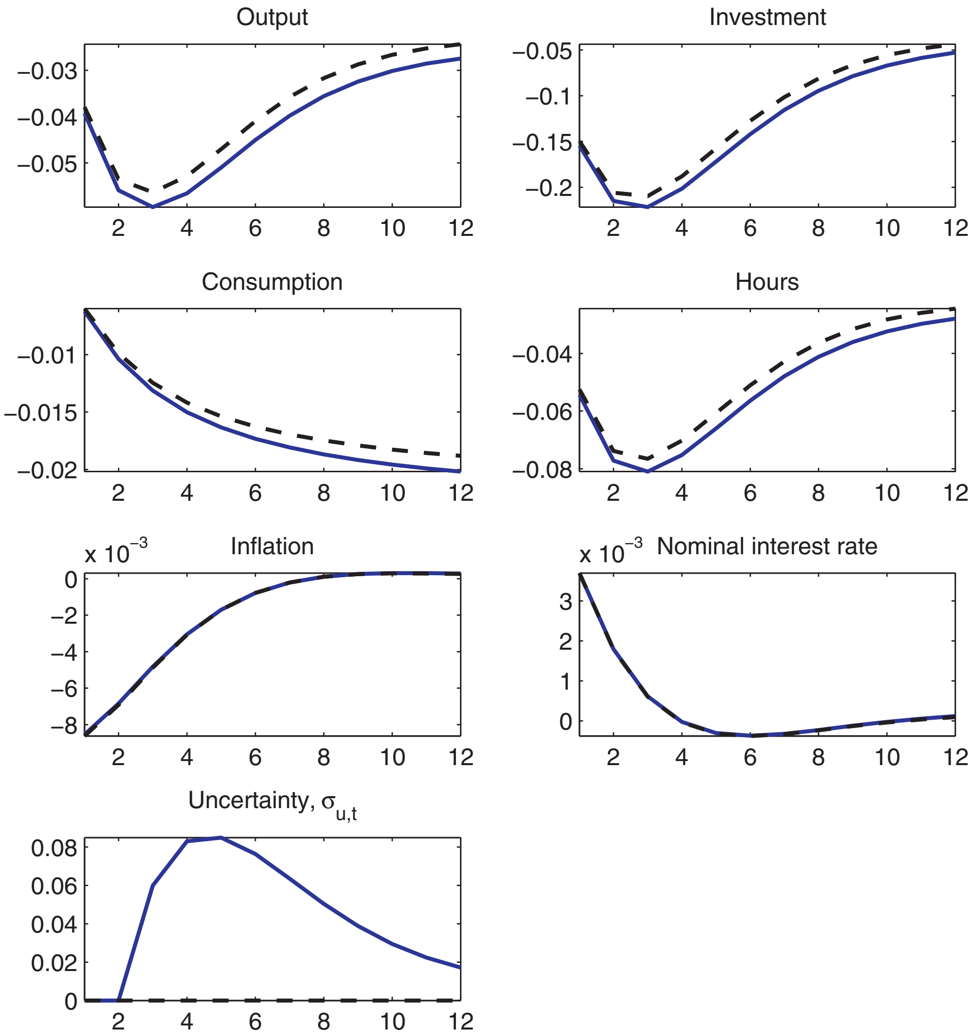

Fig. 3. Impulse response to a monetary policy shock.
Notes: Blue solid lines are the baseline and black dashed lines are when the uncertainty multiplier is turned off. Since third-order approximations move the ergodic distribution of endogenous variables away from the steady state (Fernández-Villaverde et al. 2011), I report the impulse responses in terms of percent deviation from the ergodic mean. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

"Persistent depreciation"). The sizes of the amplification are very similar to the baseline model with an i.i.d. depreciation shock.[32]

Table 2 reports the output uncertainty multiplier for various parameterizations of the standard deviation of the depreciation shock, σ_δ, holding other parameters at the baseline parameterization. First, note that the relationship between the size of σ_δ and the multiplier is non-monotonic for the reason discussed in the previous section. Second, for a reasonable range of parameterizations of σ_δ, the size of the multiplier is similar to the baseline value. For example, consider $100\sigma_\delta = 0.050$. While this parameterization implies that the autocorrelation is too low and the forecast errors are too large, the uncertainty multiplier for output is 1.15.

5.2. Inspecting the mechanism

In this section, I discuss in detail how the size of the uncertainty multiplier depends on the structure of the economy and why countercyclical uncertainty leads to amplification of business cycles.

The benchmark model features several real and nominal rigidities that are absent in a basic real business cycle model. Table 5 reports the uncertainty multiplier for output under various combinations of frictions. In each experiment, I rescale all

[32] The estimated value of the persistence of the depreciation shock, ρ_δ, is 0.29. The values for other parameters are similar to the estimated values in the baseline model.

Table 5
The role of real and nominal rigidities.

Consump. habit	Investment adj. cost	Sticky price	Sticky wage	Output amplification
✓	✓	✓	✓	1.16
✓	✓		✓	1.06
✓	✓	✓		1.01
✓	✓			1.00
	✓	✓	✓	1.06
✓		✓	✓	1.07
		✓	✓	1.05
				1.00

Notes: Both data and model moments are in logs and HP-filtered ($\lambda = 1600$). For each specification, I scale $(\sigma_z, \sigma_d, \sigma_R, \sigma_\mu, \sigma_g, \sigma_\delta)$ proportionally to replicate the standard deviation of output in the data ($\sigma_Y = 1.61$).

shocks proportionally to generate the standard deviation of output that is in line with the data. I highlight two observations. First, nominal rigidities are crucial for generating sizable multipliers. The output uncertainty multiplier is 1.06 without sticky prices and 1.01 without sticky wages. Second, frictions on the real side of the economy also matter. The output uncertainty multiplier is 1.06 without consumption habit and 1.07 without the investment adjustment cost.

Why are nominal rigidities essential for amplification? To answer this question, I review the argument made by Basu and Bundick (2016) and Fernández-Villaverde et al. (2015) by comparing impulse responses to an exogenous shock to uncertainty with and without nominal rigidities.[33] The responses to an uncertainty shock are useful because they allow me to study the effects of nominal rigidities on the transmission mechanism while holding the uncertainty process identical across the two economies. Fig. 4 shows that, when both prices and wages are flexible, the effect of an uncertainty shock to economic activity is quantitatively negligible. When uncertainty increases, households cut consumption in order to increase their savings. The fall in consumption, however, induces households to increase their labor supply due to the intratemporal first-order condition[34] and as a result, leads to an increase in output. Finally, the fall in consumption and the increase in savings drive down the real interest rate. Fig. 5 shows that, with nominal rigidities, an increase in uncertainty leads to a sizable decline in output, investment, consumption, and hours. As in the case of flexible prices, an increase in uncertainty induces households to reduce consumption and to save more. Even though households wish to increase their labor supply, hours fall in equilibrium for two reasons. First, with sticky wages, wages cannot adjust to accommodate more labor supply. Second, with sticky prices, the decline in aggregate demand leads to an increase in firms' markups and thus firms demand less labor. On the saving side, the physical capital becomes a worse hedge for aggregate shocks because the return on capital is subject to more uncertainty. Moreover, the increase in markups reduces firms' capital demand. These two factors cause investment to drop, which further reduces aggregate demand. The overall outcome is that the decline in aggregate demand and an increase in markups reinforce each other and output drops substantially.

An interesting feature of the baseline model is that, compared to quantities such as output and consumption, nominal variables such as the inflation and the nominal interest rate do not move much. The inflation is not very responsive simply because prices are sticky and most firms cannot adjust their prices in the short run. The nominal interest rate, in turn, does not move much because the Taylor rule features inertia and reacts to the inflation. Since the nominal rate declines more than the inflation, the real interest rate also falls but only by a small amount. Another way to understand the modest movement in the real rate is as follows. On one hand, the fall in investment raises the marginal return on capital. On the other hand, the increase in markups reduce the return on capital. The latter effect dominates on net and the real return on capital falls slightly, which translates into a small decline in the real rate.

That nominal rigidities are crucial for amplification implies that the size of the uncertainty multiplier also depends on the monetary policy rule in place.[35] To illustrate this, in Fig. 5 I plot the responses to an uncertainty shock when the central bank responds less aggressively to inflation ($\phi_\pi = 1.9$ instead of $\phi_\pi = 2$, green circled lines) and more aggressively to inflation ($\phi_\pi = 2.1$ instead of $\phi_\pi = 2$, red dots). The figure shows that the more accommodative the monetary policy rule (lower ϕ_π), the larger the effects of the uncertainty shock. The intuition is simple. When the nominal interest rate responds less (more) aggressively to inflation, the real interest rate decrease in response to an increase in uncertainty is smaller (larger). When $\phi_\pi = 1.9$ and $\phi_\pi = 2.1$, the uncertainty multiplier for output is 1.19 and 1.09, respectively. Similar results hold when I change the output growth coefficient in the monetary policy rule; the responses of endogenous variables to

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[33] I simulate responses of a shock to the variance $\sigma^2_{u,t}$ of the innovation to the capital accumulation Eq. (10). I assume that $\sigma^2_{u,t}$ follows $\ln\ \sigma^2_{u,t} = 0.95\ \ln\ \sigma^2_{u,t-1} + \epsilon_{\sigma,t}$ and consider a 30% innovation to $\epsilon_{\sigma,\,t}$.

[34] The argument is originally made by Barro and King (1984), who pointed out that in flexible-price models featuring time-separable preferences, consumption and hours move in the opposite direction in response to a non-technology shock.

[35] In a related analysis, Born and Pfeifer (2014) find that the effects of policy uncertainty shocks depend on the parameterization of the monetary policy rule.

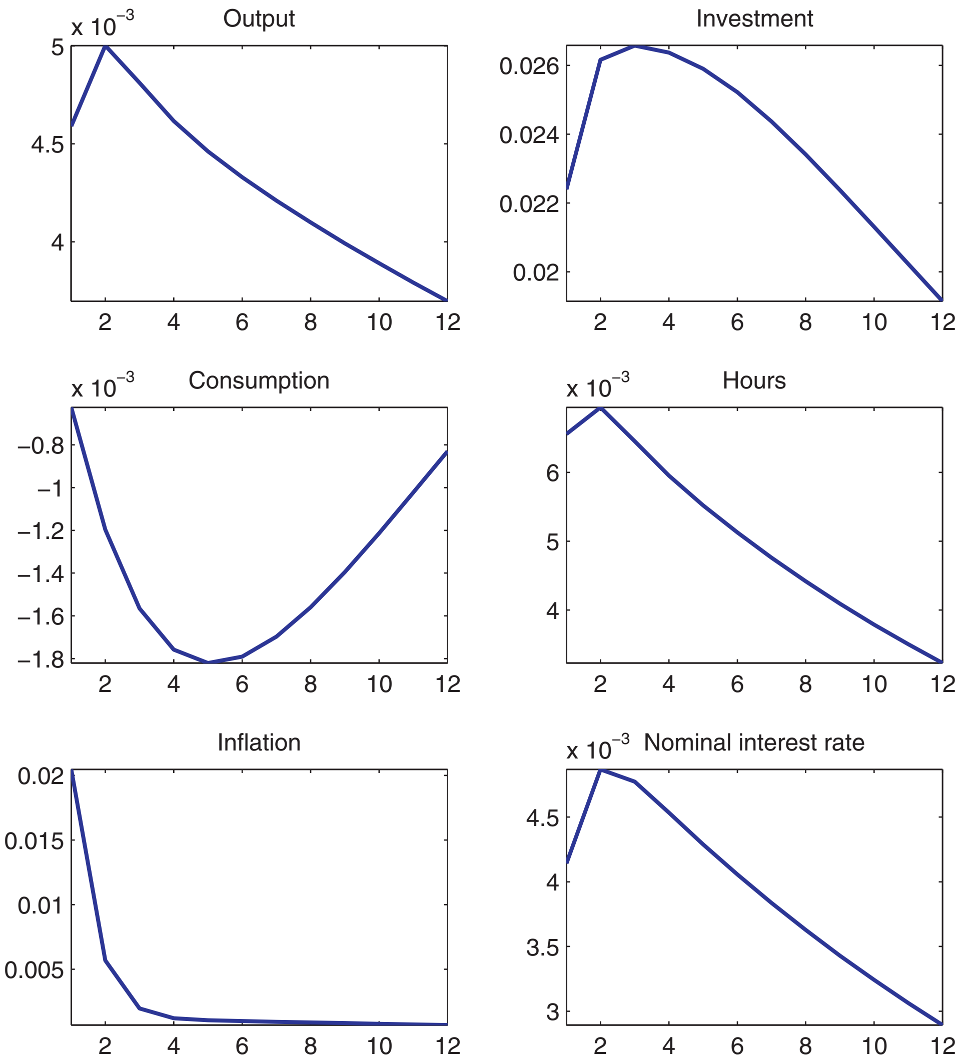

Fig. 4. Impulse response to an uncertainty shock: no nominal rigidities.
Notes: The figure shows the impulse response (in terms of percent deviation from the ergodic mean) to an uncertainty shock when both sticky prices and wages are turned off.

changes in uncertainty are larger when ϕ_Y is smaller. When $\phi_Y = 0.05$ and $\phi_Y = 0.15$, the uncertainty multiplier for output is 1.23 and 1.15, respectively.

We now discuss why real rigidities matter for amplification. Fig. 6 shows that the effect of an uncertainty shock is larger without real rigidities (green circled lines), simply because there is less friction and hence the economy is more elastic to shocks. Why then, is the uncertainty multiplier smaller without real rigidities? Recall that when I calculate the uncertainty multiplier under various combinations of frictions in Table 5, I rescale the size of shocks so that the model replicates the output standard deviation in the data. When the real rigidities are turned off, output and other economic activities respond very strongly to aggregate shocks and hence the sizes of the shocks, including the investment shock, have to be quite small in order to match the data. In turn, this implies that uncertainty about the marginal efficiency of investment is also small to begin with, leading to small output amplification.

To summarize, nominal rigidities and countercyclical markups are important for magnifying the effects of endogenous uncertainty. This finding echoes the argument made by previous papers that study uncertainty shocks in a sticky-price model. Importantly, real rigidities also increase the size of the amplification because the rigidities affect the inference about the underlying shock processes. This is in sharp contrast to the case of uncertainty shocks, where real rigidities reduce the overall effect.

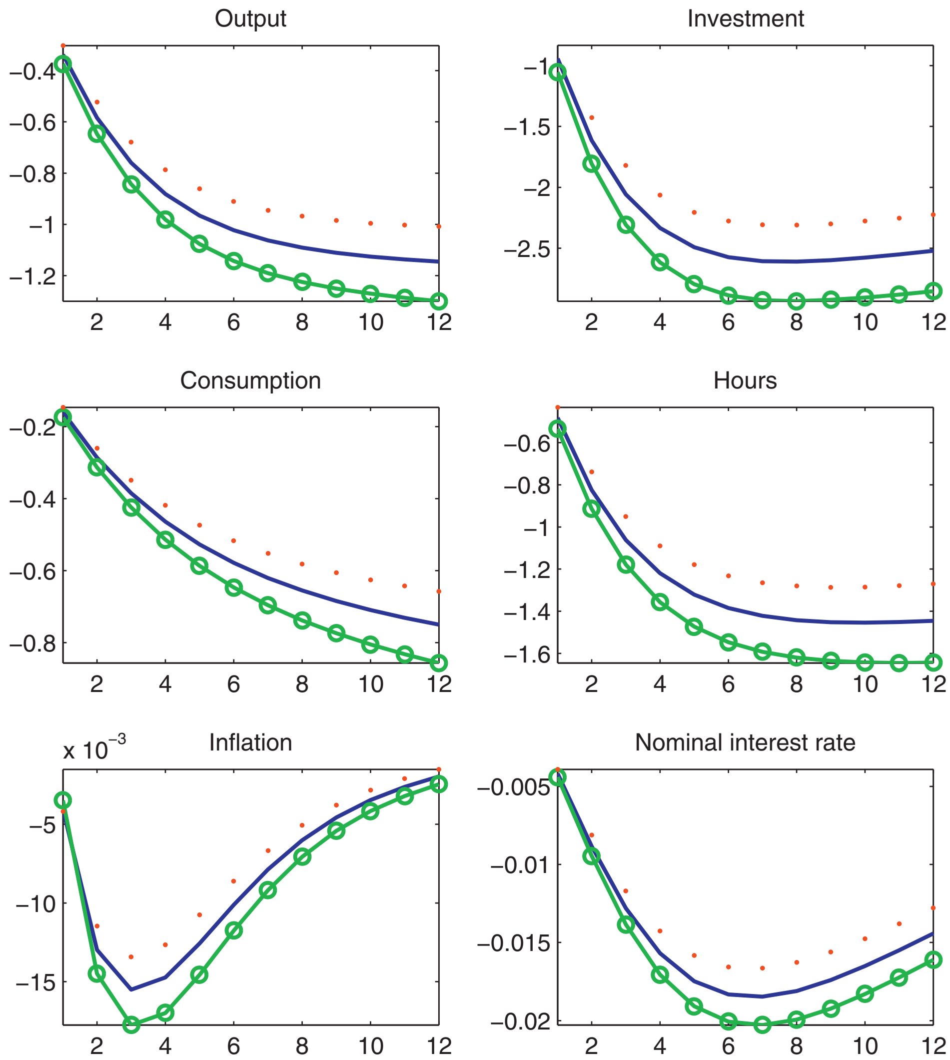

Fig. 5. Impulse response to an uncertainty shock: baseline model.
Notes: The blue solid lines are the impulse response (in terms of percent deviation from the ergodic mean) to an uncertainty shock in the baseline model ($\phi_\pi = 2$). The green circled lines are when $\phi_\pi = 1.9$ and the red dots are when $\phi_\pi = 2.1$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 6
The uncertainty multiplier is increasing in the relative size of the growth shock.

	$Corr(FE_t^{1Q}, FE_{t-1}^{1Q})$	$Mean(\lvert FE_t^{1Q}\rvert)$	Output amplification
Data	0.17	0.55	
Model			
$\sigma_\mu/\sigma_g = 1.00$	0.13	0.66	1.159
$\sigma_\mu/\sigma_g = 0.35$	0.17	0.55	1.165
$\sigma_\mu/\sigma_g = 0.00$	0.18	0.53	1.199

Notes: Both data and model moments are in logs and HP-filtered ($\lambda = 1600$). σ_μ and σ_g are scaled so that the standard deviation of output is the same as that in the benchmark specification ($\sigma_Y = 1.61$).

5.3. Changing the parameters of the shock processes

I consider the effects of changing the parameters of the shock processes from the benchmark calibration. The exercise provides additional insights regarding determinants of the size of the uncertainty multiplier.

Table 6 reports the uncertainty multiplier for output under different parameterizations of the standard deviation of the investment shock σ_μ and the growth shock σ_g. I change the ratio of the standard deviations, σ_μ/σ_g, from the benchmark

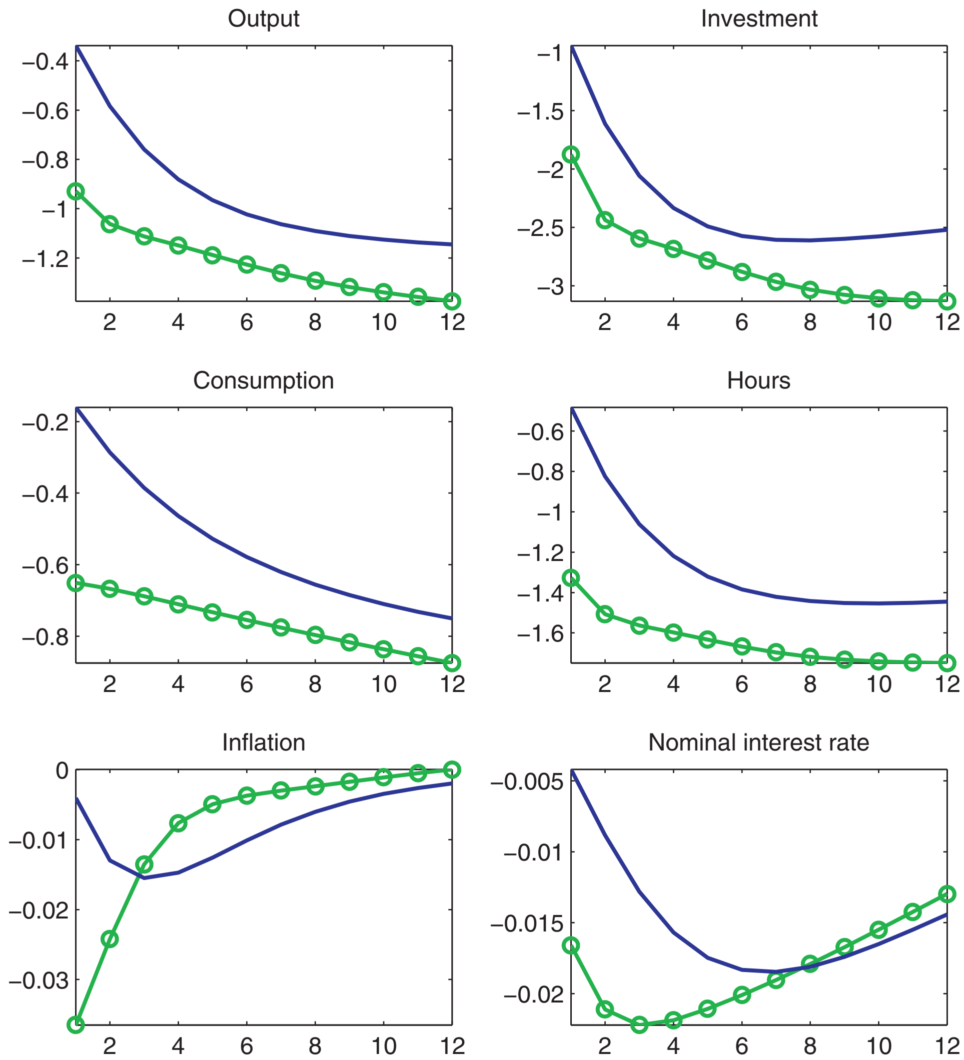

Fig. 6. Impulse response to an uncertainty shock: with and without real rigidities.
Notes: The blue solid lines are the impulse response (in terms of percent deviation from the ergodic mean) to an uncertainty shock in the baseline model and the green circled lines are the responses when real rigidities (consumption habit and investment adjustment cost) are turned off. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.).

calibration ($\sigma_\mu/\sigma_g = 0.35$) while keeping the standard deviation of output constant. The multiplier is increasing in the relative size of the growth shock. Intuitively, agents respond more to changes in uncertainty about the expected trend growth than to those about the fluctuation around the trend. The uncertainty multiplier is also increasing in the absolute size of the shocks. This can be seen in Table 7, where I scale the standard deviations of shocks (σ_z, σ_d, σ_R, σ_μ, σ_g, and σ_δ) proportionally from the benchmark calibration. The reason is that the fluctuation in uncertainty becomes more important to agents' decision making as the volatility of shocks becomes larger.

The results in this subsection have an interesting implication for emerging market economies. As shown in Aguiar and Gopinath (2007), these economies feature more volatile business cycles that could be well characterized by fluctuations in expected growth rates.[36] This suggests that the uncertainty multiplier may be much larger in emerging markets than in the U.S.

6. Empirical evidence

In this section, I confront the model with data regarding uncertainty and economic activities. First, I ask how much of the fluctuations in uncertainty that we observe in the data can be generated by the model. Second, I ask whether the model can

[36] See also Boz et al. (2011), who extend Aguiar and Gopinath (2007)'s analysis by incorporating a learning problem.

Table 7
The uncertainty multiplier is increasing in the size of shocks.

	Output standard dev.	Output amplification
Data	1.61	
Model		
$\sigma \times 0.9$	1.28	1.13
$\sigma \times 1.0$	1.61	1.16
$\sigma \times 1.1$	2.43	1.33

Notes: Both data and model moments are in logs and HP-filtered ($\lambda = 1600$). I define $\sigma \equiv (\sigma_z, \sigma_d, \sigma_R, \sigma_\mu, \sigma_g, \sigma_\delta)$.

Table 8
Log-standard deviations of uncertainty in the data and the model.

	Data	Model
SPF	14.6	6.3
VIX	33.4	7.2

Notes: Both data and model moments are in logs and multiplied by 100 to express them in percentage terms.

replicate the VAR relationship between output and uncertainty in the data. Third, I conduct Granger causality tests between output and uncertainty on both data and the model.

Table 8 compares the standard deviations of uncertainty in the data and in the model. To measure uncertainty from the data, I use probabilistic forecasts reported in the Survey of Professional Forecasters (SPF). This survey asks each forecaster for a subjective probability density of the annual percentage change in real GDP. Following the standard in the literature (Zarnowitz and Lambros 1987 and D'Amico and Orphanides 2008), I take the average across the standard deviations of those probability densities for each forecaster. The sample period is 1986:Q2–2011:Q4. Appendix A provides more details regarding the construction on the dataset. My model generates the standard deviation of 6.3% versus 14.6% in the data.[37] Intuitively, the model underpredicts the standard deviation because in the model fluctuations in uncertainty are driven mainly by fluctuations in investment while in the data the standard deviation of uncertainty is larger than that of investment by more than two-fold. As an alternative way to measure changes in uncertainty, I use the option-based expected stock market volatility index, the VIX. The sample period is 1990:Q1–2011:Q4. My model generates the standard deviation of 7.2% versus 33.4% in the data.[38] That the model can account for a smaller fraction of the data for the VIX than for the SPF may be due to the fact that the fluctuations in VIX reflect variations not only in uncertainty but also in risk aversion (Bekaert et al. 2013). Since the focus of my paper is on time-varying uncertainty, I use the SPF-based measure of uncertainty for the empirical analysis below.

As a second empirical test of the model, I characterize the relationship between real GDP and uncertainty with a generalized impulse response analysis (Pesaran and Lambros 1998) from a bivariate VAR with four lags.[39] The generalized impulse response is appealing in this context because, in contrast to a standard recursive VAR, the results are invariant to the ordering of variables. I emphasize that the purpose of this exercise is to look for a statistical relationship between output and uncertainty and hence no causal inference is drawn from the impulse responses. Fig. 7 shows that, in the data, an increase in uncertainty is associated with a gradual decline in output. On the other hand, an increase in output is associated with a decline in uncertainty. Hence, the VAR responses indicate a clear negative relationship between output and uncertainty. The figure also shows that running a VAR on the artificial data from the model generates impulse responses that are in line with the actual data. In the model, the negative relationship between output and uncertainty is due to the endogenous movement in perceived uncertainty about the efficiency of investment and its feedback to real economic activity. Note that the model replicates well the gradual responses of the two variables. This is because uncertainty is driven by investment, which exhibits hump-shaped dynamics, and this uncertainty in turn induces gradual reduction of output.

Finally, I compare the stochastic properties of the data and the model through the lens of Granger causality tests.[40] I estimate bivariate VARs (with two, three, and four lags) containing uncertainty and output and test (a) whether output Granger-causes (i.e., helps forecast) uncertainty and (b) whether uncertainty Granger-causes output.[41] Table 9 reports the p-values for the F-statistics of the Granger causality tests. The results of the model are in line with the data. For both data and the model, the hypothesis that output does not Granger-cause uncertainty are rejected at the 5% level for all the lag

[37] In the model, I define uncertainty as the standard deviation of the density forecast (conditional on the agents' information sets) of the annual percentage change in output: $Std_t(\Delta Y_{t+4})$.

[38] The model counterpart is the expected conditional standard deviation of the return on capital: $Std_t(\tilde{R}^k_{t+1})$.

[39] Both variables are logged with linear trends.

[40] This analysis is inspired by Bachmann et al. (2016), who run Granger causality tests on firm-level risk and a recession indicator using German data.

[41] As in the VAR analysis, both variables are logged with linear trends.

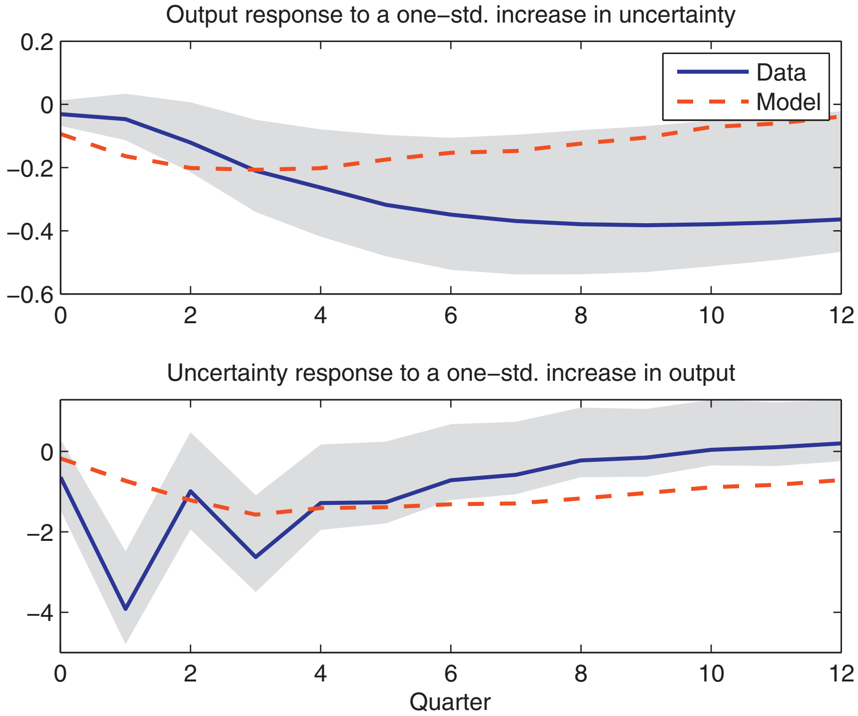

Fig. 7. Impulse responses in a bivariate VAR.
Notes: The shaded area represents ± one-standard-deviation bootstrap confidence bands.

Table 9
Uncertainty and output: Granger causality tests.

	Output Granger-causes uncertainty?		*Uncertainty Granger-causes output?*	
	Data	Model	Data	Model
VAR(2)	0.01	0.00	0.24	0.10
VAR(3)	0.01	0.00	0.67	0.10
VAR(4)	0.02	0.00	0.58	0.11

Notes: The table reports the p-values for the F-statistics of the Granger causality tests between uncertainty and output. I try VARs with lag lengths of two, three, and four. Higher p-values imply that the hypothesis of no Granger-causality between two variables cannot be rejected.

specifications (left panel). On the other hand, the hypothesis that uncertainty does not Granger-cause output cannot be rejected at the 10% level for both data and the model (right panel). Intuitively, in my model output has an explanatory power for the future level of uncertainty because uncertainty is driven by changes in economic activity. Since there is no exogenous impulse to uncertainty that moves economic activity, once lagged output is controlled for, lagged uncertainty is not useful in forecasting the level of output.[42]

7. Conclusion

This paper constructed a business cycle model where agents have imperfect information about the state of the economy and have to learn it through investment. Recessions are times of high uncertainty because agents invest less and hence learn less about the state of the economy. The countercyclical fluctuation in aggregate uncertainty interacts with rigidities and amplifies business cycles.

Because the level of learning is tied to the level of investment, changes in uncertainty are large and persistent. As a result, the size of the amplification is nontrivial; the uncertainty multiplier amplifies the standard deviation of output by 16–20%. The multiplier magnifies other real variables, such as investment and hours, by a similar amount.

[42] Note that the result that uncertainty does not Granger-cause output is not necessarily inconsistent with the VAR estimate above. Even if the coefficients on lagged uncertainty are not statistically different from zero, a positive innovation to uncertainty in the VAR can still lead to a decline in output through variations in lagged output and the contemporaneous correlation of the innovations.

Appendix A. Data source

The data set spans the period 1969Q1 to 2011Q4.[43] Whenever the data set is provided in monthly frequencies, I simply take the average to transform it into quarterly frequencies.

Data from the National Income and Product Accounts are downloaded from the Bureau of Economic Analysis website. Nominal GDP, nominal consumption (defined as the sum of personal consumption expenditures on nondurables and services), and nominal investment (defined as the sum of gross private domestic investment and personal consumption expenditures on durables) are divided by the civilian noninstitutional population,[44] downloaded from the Bureau of Labor Statistics (BLS hereafter) website, to convert the variables into per capita terms. I then divide them by the GDP deflator to convert them into real terms.

Working hours are measured by nonfarm business hours (available on the BLS website) divided by the population. Real wages are measured by hourly compensation in nonfarm business sectors (available on the BLS website) divided by the GDP deflator. Inflation rates are measured by changes in the GDP deflator. I use the effective federal funds rates (downloaded from the Federal Reserve Board website) to measure the nominal interest rates.

To compute the forecast error statistics, I use the median forecast of nominal GDP growth rate, downloaded from the FRB Philadelphia website. The one-period-ahead forecast error is defined as the one-period-ahead nominal GDP growth rate forecast minus the realized nominal GDP growth rate.

To compute uncertainty based on the subjective probability densities of the annual percentage change in real GDP by each forecaster in the SPF, I take the average across the standard deviations of those probability densities for each forecaster.[45] While the survey data start from 1968:Q4, concerns regarding data consistency and missing data force me to conduct the analysis using the data during the period 1986:Q2–2011:Q4.[46] Finally, since the survey asks for the percentage change in GDP between the previous and current calendar year, there is a seasonality in the forecast horizons. For example, in the first quarter, it is a 4-quarter-ahead forecast. In the second quarter, it is a 3-quarter-ahead forecast. I eliminate this seasonality by applying the Tramo-Seats filter.[47]

Appendix B. Countercyclical uncertainty: full derivation

I restate agents' Kalman-filtering problem below:

$$\begin{bmatrix}\mu_t\\ g_t\end{bmatrix} = \begin{bmatrix}(1-\rho_\mu)\mu\\ 0\end{bmatrix} + \begin{bmatrix}\rho_\mu & 1\\ 0 & \rho_g\end{bmatrix}\begin{bmatrix}\mu_{t-1}\\ g_{t-1}\end{bmatrix} + \begin{bmatrix}\epsilon_{\mu,t}\\ \epsilon_{g,t}\end{bmatrix},$$

$$K_t - (1-\delta)K_{t-1} = [I_{t-1} \quad 0]\begin{bmatrix}\mu_t\\ g_t\end{bmatrix} + K_{t-1}\epsilon_{\delta,t}.$$

At the end of period $t-1$, agents forecast the values of $\{\mu_t, g_t\}$:

$$\tilde{\mu}_{t|t-1} = (1-\rho_\mu)\mu + \rho_\mu\tilde{\mu}_{t-1} + \tilde{g}_{t-1},$$

$$\tilde{g}_{t|t-1} = \rho_g g_{t-1}.$$

The elements of the associated forecasting error covariance matrix, $\Sigma_{t|t-1}$, are

$$\begin{aligned}
\Sigma^{11}_{t|t-1} &= \rho_\mu^2\Sigma^{11}_{t-1} + 2\rho_\mu\Sigma^{12}_{t-1} + \Sigma^{22}_{t-1} + \sigma_\mu^2,\\
\Sigma^{12}_{t|t-1} &= \rho_\mu\rho_g\Sigma^{12}_{t-1} + \rho_g\Sigma^{22}_{t-1},\\
\Sigma^{21}_{t|t-1} &= \Sigma^{12}_{t|t-1},\\
\Sigma^{22}_{t|t-1} &= \rho_g^2\Sigma^{22}_{t-1} + \sigma_g^2.
\end{aligned}$$

After observing period t realization of capital, K_t, agents update their belief according to

$$\tilde{\mu}_t = \tilde{\mu}_{t|t-1} + \frac{I_{t-1}\Sigma^{11}_{t|t-1}}{I_{t-1}^2\Sigma^{11}_{t|t-1} + K_{t-1}^2\sigma_\delta^2}\cdot\{K_t - (1-\delta)K_{t-1} - I_{t-1}\tilde{\mu}_{t-1}\},$$

$$\tilde{g}_t = \tilde{g}_{t|t-1} + \frac{I_{t-1}\Sigma^{12}_{t|t-1}}{I_{t-1}^2\Sigma^{11}_{t|t-1} + K_{t-1}^2\sigma_\delta^2}\cdot\{K_t - (1-\delta)K_{t-1} - I_{t-1}\tilde{\mu}_{t-1}\}.$$

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[43] I pick this starting date because the Survey of Professional Forecasters began around that time.

[44] Since raw population data display occasional breaks due to changes in population controls, I use an HP-filtered ($\lambda = 1600$) trend instead.

[45] The survey asks each forecaster to place probabilities in bins spanning a wide range of outcomes for the percentage change in real GDP. To compute the individual standard deviations, I fit a normal distribution to the individual probabilities. For more details, see D'Amico and Orphanides (2008). I have also tried other methods and obtained similar results.

[46] Nevertheless I conducted the analysis using the whole sample period and found similar results.

[47] Since the survey response between the current and the following year is also available, it is possible to construct uncertainty data with different forecast horizons. I have conducted the analysis with different forecast horizons and found similar results.

The elements of the forecasting error covariance matrix are given by

$$\Sigma_t^{11} = \left[1 - \frac{I_{t-1}^2 \Sigma_{t|t-1}^{11}}{I_{t-1}^2 \Sigma_{t|t-1}^{11} + K_{t-1}^2 \sigma_\delta^2}\right] \Sigma_{t|t-1}^{11},$$

$$\Sigma_t^{12} = \left[1 - \frac{I_{t-1}^2 \Sigma_{t|t-1}^{11}}{I_{t-1}^2 \Sigma_{t|t-1}^{11} + K_{t-1}^2 \sigma_\delta^2}\right] \Sigma_{t|t-1}^{12},$$

$$\Sigma_t^{21} = \Sigma_t^{12},$$

$$\Sigma_t^{22} = \Sigma_{t|t-1}^{22} - \frac{I_{t-1}^2 (\Sigma_{t|t-1}^{12})^2}{I_{t-1}^2 \Sigma_{t|t-1}^{11} + K_{t-1}^2 \sigma_\delta^2}.$$

Thus, the elements of Σ_t are decreasing in $\frac{I_{t-1}}{K_{t-1}}$.

Appendix C. Capital accumulation in a decentralized equilibrium

To implement capital accumulation and accommodate both investment-specific technology (IST) and marginal efficiency of investment (MEI) shocks in a decentralized competitive equilibrium, we introduce perfectively competitive investment goods producers and capital goods producers owned by the households (Justiniano et al. 2011).

Investment goods producers purchase Y_t^I units of final consumption goods at price P_t and transform and transform them into I_t units of investment goods, which they sell to capital producers at price P_t^I. They choose the level of inputs Y_t^I to maximize the profit

$$\max_{Y_t^I} P_t^I I_t - P_t Y_t^I,$$

subject to the production function

$$I_t = \nu_t Y_t^I,$$

where ν_t is the IST shock that follows

$$\nu_t = (1 - \rho_\nu)\nu + \rho_\nu \nu_{t-1} + \epsilon_{\nu,t},$$

where $\epsilon_{\nu,t}$ is i.i.d. distributed from a normal distribution with mean zero and variance σ_ν^2. In equilibrium, the relative price of investment goods to consumption goods is inversely related to the IST:

$$\frac{P_t^I}{P_t} = \frac{1}{\nu_t}.$$

At the end of each period t, the capital producers purchase investment goods I_t from the investment goods producers and capital K_t from households at price $\tilde{Q}_t$. In period $t+1$, they build new capital K_{t+1} using the technology (8). The capital producers can observe the path of capital stock and investment but cannot observe the underlying shocks. The new capital is sold at price Q_{t+1}. The profits are transferred back in a lump-sum manner each period. The capital producers choose the inputs I_t and K_t to maximize their discounted sum of profits:

$$\max_{I_t, K_t} -\lambda_t (P_t^I I_t + \tilde{Q}_t K_t) + \mathrm{E}_t \sum_{s=0}^{\infty} \beta^{s+1} \lambda_{t+s+1}$$
$$\times \left[Q_{t+s+1} \left\{ (1 - \delta_{t+s+1}) K_{t+s} + \mu_{t+s+1} \left(1 - S\left(\frac{I_{t+s}}{I_{t+s-1}} \right) \right) I_{t+s} \right\} - P_{t+s+1}^I I_{t+s+1} - \tilde{Q}_{t+s+1} K_{t+s+1} \right].$$

The first-order conditions of this profit maximization problem yield an evolution for the expected price of capital:

$$\lambda_t P_t^I = \beta \mathrm{E}_t \left[\lambda_{t+1} Q_{t+1} \mu_{t+1} \left\{ 1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} \right\} + \beta \lambda_{t+2} Q_{t+2} \mu_{t+2} S'\left(\frac{I_{t+1}}{I_t}\right) \left(\frac{I_{t+1}}{I_t}\right)^2 \right].$$

They also provide an expression for the “rental” rate of capital:

$$\lambda_t \tilde{Q}_t = \beta \mathrm{E}_t [(1 - \delta_t) \lambda_{t+1} Q_{t+1}],$$

which says that the price of capital at the end of period takes into account the discounting and depreciation that occur at the beginning of the next period. Note that the price of capital, Q_t, do not reveal additional information about the unobservable shocks. This is simply because capital producers also face the filtering problem described in Section 2 in the main text.

As in the baseline model, I parameterize the extended model in two steps. First, I fix parameters as in Table 1. Second, I use the SMM to estimate the parameters for the IST shock ρ_ν and σ_ν and the 9 parameters that were estimated in the baseline model. In addition to the 9 moments (macro and survey data moments) that were used in the baseline model, I match the autocorrelation and standard deviation of the relative price of investment.[48] Including the moments of the relative price of investment helps me identify the parameters for the IST shock. The results of the extended model are similar to the baseline model although the sizes of the amplification are slightly smaller. For example, the output, investment, and house amplifications are 13%, 12%, and 14%, respectively.

Appendix D. Procyclical signal-to-noise ratio arising from aggregation

The procyclical signal-to-noise ratio arises from aggregation of investment units with common and idiosyncratic shocks. The argument does not require depreciation shocks and closely follows the discussion made in Van Nieuwerburgh and Veldkamp (2006).

Consider an economy with many investment units, where each unit has a technology that transforms investment goods into efficiency units of capital. The capital production is increasing in the number of investment units operating. Denote N_t as the number of investment units operating at time t. The output of each unit is the product of its own efficiency, which has a common component μ_t and idiosyncratic component η_t^i, and its input normalized to $i_t^i = 1$. Then, aggregate net investment is the sum of output of all the investment units:

$$K_t - (1-\delta)K_{t-1} = \sum_{i=1}^{N_t} (\mu_t + \eta_t^i) i_t^i = \mu_t N_t + \sum_{i=1}^{N_t} \eta_t^i$$

Define the signal-to-noise ratio as $\mathrm{Var}(\mu_t N_t)/\mathrm{Var}(\sum_{i=1}^{N_t} \eta_t^i)$. Then, as long as the correlation of idiosyncratic shocks across investment units is less than one, the signal-to-noise ratio is increasing in the number of units operating, N_t. A more detailed proof of this argument can be found in Van Nieuwerburgh and Veldkamp (2006).

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