Redistribution and Fiscal Uncertainty Shocks
Online Appendix

This Online Appendix is organized as follows. In Section A, I compare the smoothed fiscal volatility and other measures of fiscal uncertainty. In Section B, I provide additional results regarding household-level impulse responses to fiscal uncertainty shocks. Section C lists the equilibrium conditions and Section D describes the solution method. In Section E, I compute the Euler Equation Error to evaluate the accuracy of the approximated solution. In Section F, I describe the Bayesian impulse-response-matching method that is used to estimate the model. In Section G, I report additional robustness analysis and experiments related to the main results from the paper. Finally, in Section H, I outline the data sources.

A Empirical properties of fiscal uncertainty shocks

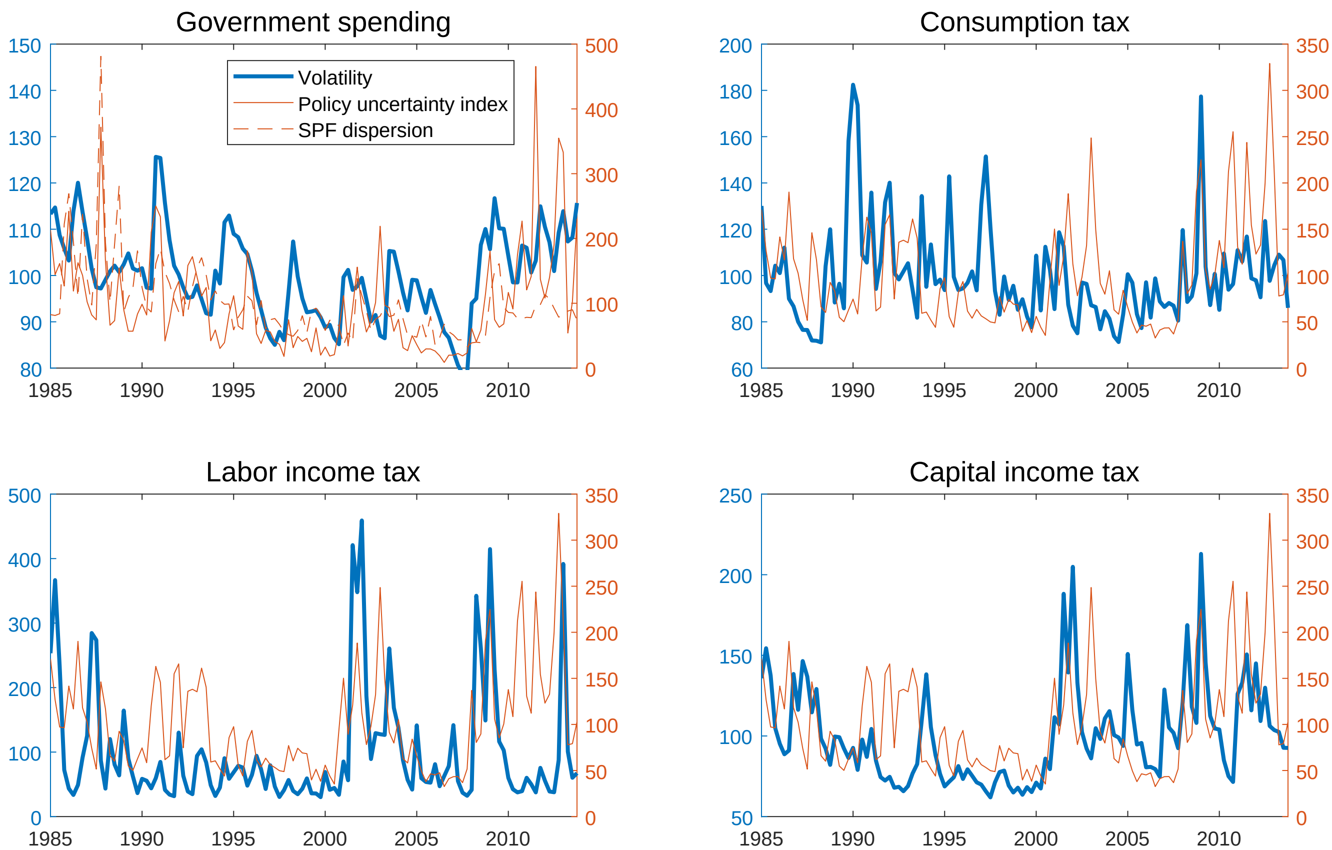
To evaluate how well the fiscal volatility captures fluctuations in fiscal uncertainty agents in the U.S. economy perceive, in Figure A1, I compare the smoothed fiscal volatility $\sigma_{x,t}$ in (2) for each fiscal instrument with other fiscal uncertainty measures.[1] For government spending volatility I compare government spending uncertainty index in Baker et al. (2016) and the interquartile range of one-quarter-ahead forecasts for real federal government spending growth from the Survey of Professional Forecasters (SPF).[2] The government spending uncertainty index is based on frequencies of newspaper articles that contain terms related to government spending. As in Ilut and Schneider (2014), the SPF dispersion can be interpreted as a measure of Knightian uncertainty (ambiguity). The logic is as follows: agents sample experts' opinions and aggregate them, according to their preferences, when making decisions. Since the decision makers are ambiguity averse, stronger disagreement among these professional forecasters generates lower confidence in the probability assessments of the future. The correlation between $\sigma_{g,t}$ and the government spending uncertainty index is 0.45 while the correlation between $\sigma_{g,t}$ and the SPF dispersion is 0.30. Both correlations are significant at a 1 percent level. For tax volatility $\sigma_{\tau_c,t}$, $\sigma_{\tau_h,t}$ and $\sigma_{\tau_k,t}$, I compare them with the tax policy uncertainty index from Baker et al. (2016), which is based on frequencies of newspaper articles that contain terms related to taxes.[3] Their correlations are 0.13 ($\sigma_{\tau_c,t}$ and tax policy uncertainty), 0.23 ($\sigma_{\tau_h,t}$ and tax policy uncertainty) and 0.27 ($\sigma_{\tau_k,t}$ and tax policy uncertainty). The correlation between average tax volatility $\bar{\sigma}_t \equiv (\sigma_{\tau_c,t} + \sigma_{\tau_h,t} + \sigma_{\tau_k,t})/3$ and the tax policy uncertainty index is 0.27. The correlation for the labor income tax volatility is significant at a 5 percent level while the correlations for the capital income tax volatility and average tax volatility are significant at a 1 percent level. These results are reassuring because they indicate that the movements in estimated fiscal volatility are in line with fluctuations in other fiscal uncertainty measures computed using very different methodologies and as a result provide an external validation of this paper's methodology to measure fiscal uncertainty.

[1] See Fernández-Villaverde et al. (2015) for a similar exercise.

[2] The sample period for this exercise is 1985:Q1–2013:Q4 since the Baker et al. (2016) index begins in 1985:Q1.

[3] The tax policy uncertainty index is not available for individual tax components. The survey data of forecasts is only available for government spending and not taxes.

Figure A1: Fiscal volatility and uncertainty

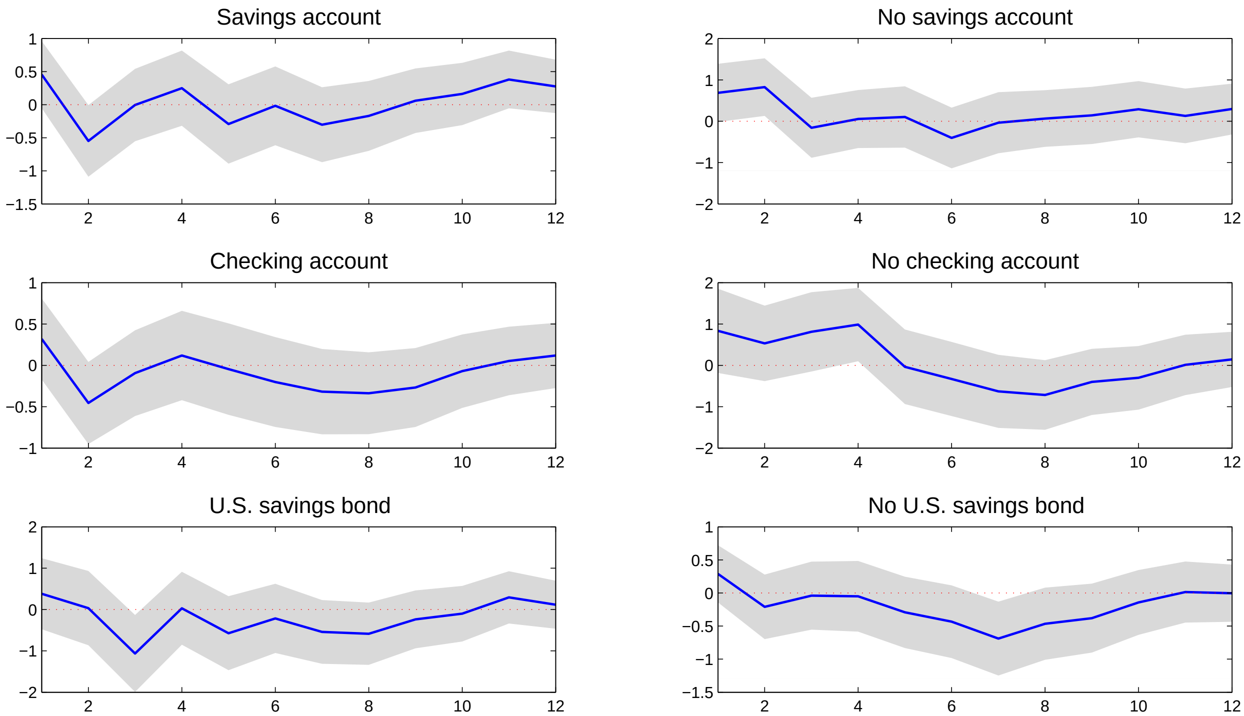


Notes: All series are normalized so that the average level is 100.

B Household-level impulse responses to fiscal uncertainty shocks: Additional results

This section provides additional results regarding the empirical household-level impulse responses to the capital income tax volatility shock. Figure A2 shows the responses of consumption for each household type, classified based on the holding statuses of assets other than stocks. This exercise allows me to check whether the heterogeneous impulse response between capital holders and non-capital holders in the main paper is driven by the holding statuses of other assets. The first row compares households who own savings accounts and those who do not. The classification is based on the responses to the question that asks about their amounts in "Savings accounts at banks, savings and loans, credit unions, etc.". The second row compares households who have checking accounts and those who do not. I identify household status using the responses to the question that asks about their amounts in "Checking accounts, brokerage accounts and other similar accounts". Finally, the third row compares households who hold U.S. savings bonds and those who do not. The savings bond holders are identified from the responses to the question that asks about their amounts in "U.S. savings bonds". In my sample (1982:Q1–2008:Q3), 55%, 73%,

Figure A2: Household-level consumption responses to a capital income tax volatility shock: splits based on holding of assets other than stocks

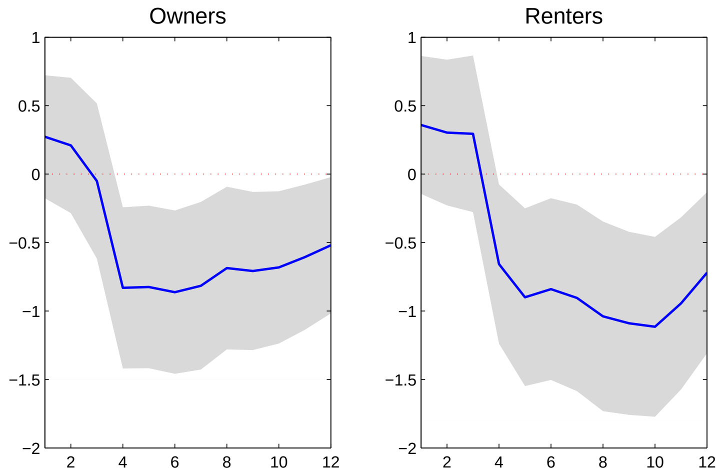


Notes: The figure reports the impulse responses to a two-standard-deviations increase in σ_{τ_k} (capital income tax volatility). The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses of CEX consumption from the VAR and the shaded areas are the 95% confidence band.

and 10% of all households own savings accounts, checking accounts, and U.S. savings bonds, respectively. Figure A2 shows that the household consumption responses are not significant at most horizons for all classifications. Hence, the heterogeneous impulse responses between capital holders and non-capital holders in the main paper are not driven by the holding statuses of other assets.

Next, I consider the household-level consumption responses to property tax volatility shocks. This exercise is interesting because it shows that even if a tax appears to be strongly redistributive, a volatility shock does not necessarily lead to heterogeneous impulse responses. To do this, I estimate the property tax and the volatility processes (equations (1) and (2) in the main paper) using a particle filter. I construct the quarterly property tax rate series by dividing the total property tax revenue (source: "Quarterly Summary of State and Local Tax Revenue" table from the Bureau of the Census) by nominal GDP. I then feed in the filtered property tax volatility shock to the baseline VAR in the main paper. However, instead of including the CEX consumption of capital holders and non-capital holders, I include the CEX consumption of home owners and renters. Figure A3 shows that in response to an increase in property tax volatility, both owners and renters reduce their consumption. Recall that in the theoretical model in the main paper, capital holders do not reduce their consumption in response to a capital income tax uncertainty shock

Figure A3: Household-level consumption responses to a property tax volatility shock: home owners vs renters



Notes: The figure reports the impulse responses to a two-standard-deviations increase in property tax volatility. The units are in percents (annual percentage points for inflation and nominal rate). The black lines are the mean responses of CEX consumption from the VAR and the shaded areas are the 95% confidence band.

because of the substitution effect: capital holders substitute away from investment in capital and increase consumption. For many home owners, it is likely that cutting residential investment is simply too costly, either because the only houses they own are their main residences or because of the transaction costs. Hence, home owners (who fear high property tax) and renters (who fear low property tax) both perceive a negative income effect and cut consumption when property tax volatility increases.

C Equilibrium conditions

In this section I report the equations that characterize the equilibrium of the estimated model presented in Section 3. First, the variables have to be scaled in order to induce stationarity. The variables are scaled as follows:

$$c_t^c = \frac{C_t^c}{\gamma^t}, i_t^c = \frac{I_t^c}{\gamma^t}, b_t^c = \frac{B_t^c}{\gamma^t}, k_{t-1}^c = \frac{K_{t-1}^c}{\gamma^t}, i_t^c = \frac{I_t^c}{\gamma^t}, t_t^c = \frac{T_t^c}{\gamma^t}, \lambda_t^c = \gamma^t \Lambda_t^c, q_t = \gamma^t Q_t,$$

$$c_t^n = \frac{C_t^n}{\gamma^t}, b_t^n = \frac{B_t^n}{\gamma^t}, t_t^n = \frac{T_t^n}{\gamma^t}, \lambda_t^n = \gamma^t \Lambda_t^n,$$

$$w_t = \frac{W_t}{\gamma^t}, k_{t-1} = \frac{K_{t-1}}{\gamma^t}, c_t = \frac{C_t}{\gamma^t}, i_t = \frac{I_t}{\gamma^t}, t_t = \frac{T_t}{\gamma^t}, w_t = \frac{W_t}{\gamma^t}, b_t^g = \frac{B_t^g}{\gamma^t},$$

where Λ_t^c, Q_t, Λ_t^n are the lagrangian multipliers for the budget constraint for the capital holders (7), the capital accumulation equation (8), and the budget constraint for non-capital holders (10), respectively. I use E_t^c and E_t^n to denote capital holders' and non-capital holders' period t conditional expectations under the worst-case belief, respectively.

Capital holders' marginal utility:

$$(1+\tau_{c,t})\lambda_t^c = \frac{1}{c_t^c - b\gamma^{-1}c_{t-1}^c} - \beta d_t E_t^c\left(\frac{b}{\gamma c_{t+1}^c - bc_t^c}\right) \tag{A1}$$

Bond decision (FONC w.r.t. B_t^c):

$$\gamma\lambda_t^c = \beta d_t E_t^c \lambda_{t+1}^c \frac{R_t}{\pi_{t+1}} \tag{A2}$$

Capital accumulation decision (FONC w.r.t. K_t^c):

$$\gamma q_t = \beta d_t E_t^c\Big[\lambda_{t+1}^c\{(1-\tau_{k,t+1})R_{t+1}^k + \tau_{k,t+1}\delta\} + q_{t+1}(1-\delta)\Big] \tag{A3}$$

with the law of motion for capital:

$$\gamma k_t^c = (1-\delta)k_{t-1}^c + \left\{1 - \frac{\kappa}{2}\left(\frac{\gamma i_t^c}{i_{t-1}^c} - \gamma\right)^2\right\} i_t^c \tag{A4}$$

Investment decision (FONC w.r.t. I_t^c):

$$\gamma\lambda_t^c = \gamma q_t\left\{1 - \frac{\kappa}{2}\left(\frac{\gamma i_t^c}{i_{t-1}^c} - \gamma\right)^2 - \kappa\left(\frac{\gamma i_t^c}{i_{t-1}^c} - \gamma\right)\frac{\gamma i_t^c}{i_{t-1}^c}\right\} + \beta d_t E_t^c q_{t+1}\kappa\left(\frac{\gamma i_{t+1}^c}{i_t^c} - \gamma\right)\left(\frac{\gamma i_{t+1}^c}{i_t^c}\right)^2 \tag{A5}$$

Conditions associated with capital holders' sticky wages:

$$f_{c,t}^1 = f_{c,t}^2, \tag{A6}$$

$$f_{c,t}^1 = (w_{c,t}^*)^{1-\theta_w}\lambda_t^c(1-\tau_{h,t})H_t w_t + \xi_w\beta d_t E_t^c\left(\frac{\pi w_{c,t}^*}{\pi_{t+1}^w w_{c,t+1}^*}\right)^{1-\theta_w} f_{c,t+1}^1, \tag{A7}$$

$$f_{c,t}^2 = \frac{\theta_w}{\theta_w - 1}(w_{c,t}^*)^{-\theta_w(1+\phi)} H_t^{1+\phi} + \xi_w\beta d_t E_t^c\left(\frac{\pi w_{c,t}^*}{\pi_{t+1}^w w_{c,t+1}^*}\right)^{-\theta_w(1+\phi)} f_{c,t+1}^2, \tag{A8}$$

$$\pi_t^w = \pi_t w_t / w_{t-1} \tag{A9}$$

Aggregate hours of capital holders is given by

$$\begin{aligned} H_t^c &= \int_0^1 \left(\frac{w_{i,c,t}}{w_t}\right)^{-\theta_w} H_t di \\ &= (w'_{c,t})^{-\theta_w} H_t, \end{aligned} \tag{A10}$$

where

$$w'_{c,t} \equiv \frac{[\int_0^1 (w_{i,c,t})^{-\theta_w} di]^{-\frac{1}{\theta_w}}}{w_t},$$

and additionally we have

$$(w'_{c,t})^{-\theta_w} = (1-\xi_w)(w_{c,t}^*)^{-\theta_w} + \xi_w\left(\frac{\pi w'_{c,t-1}}{\pi_t^w}\right)^{-\theta_w}. \tag{A11}$$

Total wages of capital holders is given by

$$\begin{aligned}\int_0^1 w_{i,c,t} H^c_{i,t} di &= \int_0^1 w_{i,c,t} \left(\frac{w_{i,c,t}}{w_t} \right)^{-\theta_w} H_t di \\ &= w_t H_t (w^\star_{c,t})^{1-\theta_w},\end{aligned} \tag{A12}$$

where

$$w^\star_{c,t} \equiv \frac{[\int_0^1 (w_{i,c,t})^{1-\theta_w} di]^{\frac{1}{1-\theta_w}}}{w_t},$$

and additionally we have

$$(w^\star_{c,t})^{1-\theta_w} = (1-\xi_w)(w^*_{c,t})^{1-\theta_w} + \xi_w \left(\frac{\pi w^\star_{c,t-1}}{\pi^w_t} \right)^{1-\theta_w}. \tag{A13}$$

Conditions that relate input demands to factor prices:

$$w_t = mc_t (1-\alpha) \frac{y_t}{h_t}, \tag{A14}$$

$$R^k_t = mc_t \alpha \frac{y_t}{k_{t-1}}, \tag{A15}$$

where mc_t is the real marginal cost.

Conditions associated with sticky prices:

$$p^*_t = \left(\frac{\theta_p}{\theta_p - 1} \right) \frac{P^n_t}{P^d_t}, \tag{A16}$$

$$P^n_t = \lambda^c_t mc_t y_t + \xi_p \beta d_t E^c_t \left(\frac{\pi_{t+1}}{\pi} \right)^{\theta_p} P^n_{t+1}, \tag{A17}$$

$$P^d_t = \lambda^c_t y_t + \xi_p \beta d_t E^c_t \left(\frac{\pi_{t+1}}{\pi} \right)^{\theta_p - 1} P^d_{t+1}, \tag{A18}$$

$$1 = (1-\xi_p)(p^*_t)^{1-\theta_p} + \xi_p \left(\frac{\pi}{\pi_t} \right)^{1-\theta_p}, \tag{A19}$$

$$\tilde{y}_t = (\tilde{p}_t)^{-\theta_p} y_t, \tag{A20}$$

$$\tilde{p}_t = (1-\xi_p)(p^*_t)^{-\theta_p} + \xi_p \left(\frac{\pi}{\pi_t} \right)^{-\theta_p}, \tag{A21}$$

where the last two equations are the aggregation due to Calvo pricing.

Non-capital holders' marginal utility:

$$(1+\tau_{c,t})\lambda^n_t = \frac{1}{c^n_t - b\gamma^{-1} c^n_{t-1}} - \beta d_t E^n_t \left(\frac{b}{\gamma c^n_{t+1} - b c^n_t} \right) \tag{A22}$$

Bond decision (FONC w.r.t. B^n_t):

$$\gamma \lambda^n_t \left(1 + \upsilon \frac{b^n_t}{y_t} \right) = \beta d_t E^n_t \lambda^n_{t+1} \frac{R_t}{\pi_{t+1}} \tag{A23}$$

Non-capital holders' budget constraint:

$$(1+\tau_{c,t})c_t^n + b_t^n = (1-\tau_{h,t})\int_0^1 w_{i,n,t}H_{i,t}^n di + R_{t-1}\frac{b_{t-1}^n}{\gamma\pi_t} + t_t - \frac{v}{2}\left(\frac{b_t^n}{y_t}\right)^2 y_t \tag{A24}$$

Conditions associated with non-capital holders' sticky wages:

$$f_{n,t}^1 = f_{n,t}^2, \tag{A25}$$

$$f_{n,t}^1 = (w_{n,t}^*)^{1-\theta_w}\lambda_t^n(1-\tau_{h,t})H_t w_t + \xi_w\beta d_t E_t^n\left(\frac{\pi w_{n,t}^*}{\pi_{t+1}^w w_{n,t+1}^*}\right)^{1-\theta_w} f_{n,t+1}^1, \tag{A26}$$

$$f_{n,t}^2 = \frac{\theta_w}{\theta_w-1}(w_{n,t}^*)^{-\theta_w(1+\phi)}H_t^{1+\phi} + \xi_w\beta d_t E_t^n\left(\frac{\pi w_{n,t}^*}{\pi_{t+1}^w w_{n,t+1}^*}\right)^{-\theta_w(1+\phi)} f_{n,t+1}^2 \tag{A27}$$

Aggregate hours of capital holders is given by

$$\begin{aligned} H_t^n &= \int_0^1 \left(\frac{w_{i,n,t}}{w_t}\right)^{-\theta_w} H_t di \\ &= (w_{n,t}')^{-\theta_w} H_t, \end{aligned} \tag{A28}$$

where

$$w_{n,t}' = \frac{[\int_0^1 (w_{i,n,t})^{-\theta_w} di]^{-\frac{1}{\theta_w}}}{w_t},$$

and additionally we have

$$(w_{n,t}')^{-\theta_w} = (1-\xi_w)(w_{n,t}^*)^{-\theta_w} + \xi_w\left(\frac{\pi w_{n,t-1}'}{\pi_t^w}\right)^{-\theta_w}. \tag{A29}$$

Total wages of capital holders is given by

$$\begin{aligned} \int_0^1 w_{i,n,t}H_{i,t}^n di &= \int_0^1 w_{i,n,t}\left(\frac{w_{i,n,t}}{w_t}\right)^{-\theta_w} H_t di \\ &= w_t H_t (w_{n,t}^\star)^{1-\theta_w}, \end{aligned} \tag{A30}$$

where

$$w_{n,t}^\star = \frac{[\int_0^1 (w_{i,n,t})^{1-\theta_w} di]^{\frac{1}{1-\theta_w}}}{w_t},$$

and additionally we have

$$(w_{n,t}^\star)^{1-\theta_w} = (1-\xi_w)(w_{n,t}^*)^{1-\theta_w} + \xi_w\left(\frac{\pi w_{n,t-1}^\star}{\pi_t^w}\right)^{1-\theta_w}. \tag{A31}$$

Production function:

$$\tilde{y}_t = k_{t-1}^\alpha h_t^{1-\alpha} \tag{A32}$$

Monetary policy rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_R}\left\{\left(\frac{\pi_t}{\bar{\pi}}\right)^{\phi_\pi}\left(\frac{y_t}{\bar{Y}}\right)^{\phi_Y}\right\}^{1-\rho_R} \tag{A33}$$

Government budget constraint:

$$t_t = b_t^g - \frac{R_{t-1}}{\gamma\pi_t}b_{t-1}^g + \tau_{c,t}c_t + \tau_{h,t}\left[\chi\int_0^1 w_{i,n,t}H_{i,t}^n di + (1-\chi)\int_0^1 w_{i,c,t}H_{i,t}^c di\right] + \tau_{k,t}(R_t^k - \delta)k_{t-1} - g_t y_t \tag{A34}$$

Government debt process:

$$\hat{b}_t^g = \rho_B\hat{b}_{t-1}^g + \phi_{B,Y}\hat{y}_{t-1} + \phi_{B,T}\hat{t}_{t-1} \tag{A35}$$

Aggregation:

$$c_t = \chi c_t^n + (1-\chi)c_t^c, \tag{A36}$$

$$H_t = \chi H_t^n + (1-\chi)H_t^c, \tag{A37}$$

$$i_t = (1-\chi)i_t^c, \tag{A38}$$

$$k_t = (1-\chi)k_t^c. \tag{A39}$$

Resource constraint:

$$c_t + i_t + g_t y_t + \frac{v}{2}\left(\frac{b_t^n}{y_t}\right)^2 y_t = y_t \tag{A40}$$

Bond-market clearing condition:

$$\chi b_t^n + (1-\chi)b_t^c = b_t^g \tag{A41}$$

The 41 endogenous variables we solve are

$$y_t, c_t^c, c_t^n, c_t, \lambda_t^c, \lambda_t^n, q_t, k_t^c, k_t, i_t^c, i_t, H_t^c, H_t^n, H_t, f_{c,t}^1, f_{c,t}^2, w_{c,t}^*, \pi_t^w, w_{c,t}', w_{c,t}^\star,$$

$$f_{n,t}^1, f_{n,t}^2, w_{c,t}^*, w_{c,t}', w_{c,t}^\star, b_t^c, b_t^n, w_t, R_t^k, R_t, mc_t, p_t^*, p_t^n, p_t^d, \pi_t, \tilde{y}_t, \tilde{p}_t, t_t, b_t^g, \int_0^1 w_{i,c,t}H_{i,t}^c di, \int_0^1 w_{i,n,t}H_{i,t}^n di.$$

We have listed 41 equilibrium conditions above from (A1) to (A41).

D Solution method

In this section, I first describe how to compute the solution of the model when the economy is not in the ZLB. In the second part, I describe how to derive the solution under the ZLB. For this I utilize the procedure by Cagliarini and Kulish (2013). Although the first part closely follows Ilut et al. (2016), I include it in the interest of completeness and because it is useful to understand the solution under the ZLB.

Let Y_t denote a $n \times 1$ vector of endogenous variables and Z_t a $k \times 1$ vector of exogenous state variables. The system of equilibrium conditions are composed of three types of equations. The first are the equations that describe the evolution of endogenous variables but do not involve expectations:

$$f(Y_t, Y_{t-1}, Z_t) = 0$$

Then there are equations that involves expectations. I explicitly distinguish between different agents' belief sets on which expectations are based. Suppose there are m_i equations for each agent $i = c, n$ (capital-holders and non-capital holders). Then there are total of $\sum_i m_i = m$ equations:

$$E_t^i[g^i(Y_t, Y_{t-1}, Y_{t+1}, Z_t)] = 0$$

Finally, there are k equations that characterize the law of motion of exogenous variables:

$$\ln Z_t = (I - P) \ln \bar{Z} + P \ln Z_{t-1} + \varepsilon_t$$

Agents' expected Z_{t+1} under the worst-case belief is given by

$$E_t^i \ln Z_{t+1} = (I - P) \ln \bar{Z} + P \ln Z_t + A^i \ln Z_t$$

where the matrix A^i determines the belief adjustment relative to the true law of motion above. In my model, Z_t contains the bound $a_{x,t}$ for each fiscal instruments and hence I use different matrices A^i to pick different worst-case scenario for each agent.

To compute the solution of the model outside the ZLB, we follow the steps below:

1. Guess the elasticities, ε_{yy} and ε_{yz}, of endogenous variables Y with respect to endogenous and exogenous variables, respectively.

2. Compute the candidate steady state $\bar{Y}$ by evaluating the equations

$$\begin{aligned} f(\bar{Y}, \bar{Y}, \bar{Z}) &= 0 \\ g^i(\bar{Y}, \bar{Y}, \bar{Y} \exp(\epsilon_{yz} A^i \ln \bar{Z}), \bar{Z}) &= 0 \end{aligned}$$

3. Solve for the elasticities at the steady state by considering the log-linearized equilibrium conditions

$$G^0 \hat{Y}_t = G^1 \hat{Y}_{t-1} + G^2 E_t^i \hat{Y}_{t+1} + \Psi \hat{Z}_t$$

To solve for the worst-case expectations, we use

$$E_t^i \hat{Y}_{t+1} = \varepsilon_{yy} \hat{Y}_t + \varepsilon_{yz} (P + A^i) \hat{Z}_t$$

and hence we have

$$G^0 \hat{Y}_t = G^1 \hat{Y}_{t-1} + G^2 \varepsilon_{yy} \hat{Y}_t + [G^2 \varepsilon_{yz} (P + A^i) + \Psi] \hat{Z}_t$$

The solution is given by

$$\hat{Y}_t = \tilde{\varepsilon}_{yy} \hat{Y}_{t-1} + \tilde{\varepsilon}_{yz} \hat{Z}_t$$

where, using undetermined coefficients,

$$\tilde{\varepsilon}_{yy} = (G^0 - G^2\varepsilon_{yy})^{-1}G^1$$
$$\tilde{\varepsilon}_{yz} = (G^0 - G^2\varepsilon_{yy})^{-1}[G^2\varepsilon_{yz}(P + A^i) + \Psi]$$

4. Check that the resulting elasticities, $\tilde{\varepsilon}_{yy}$ and $\tilde{\varepsilon}_{yz}$, coincide with the initial guesses, ε_{yy} and ε_{yz}. If not, set the new guesses to $\varepsilon_{yy} = \tilde{\varepsilon}_{yy}$ and $\varepsilon_{yz} = \tilde{\varepsilon}_{yz}$ and return to step 1.

5. Verify that agents are indeed forming expectations under the worst-case beliefs. This can be done by checking that the expected continuation value for each agent, $E^c_t V^c_{t+1}$ and $E^n_t V^n_{t+1}$, decreases as uncertainty for each fiscal instrument increases.

I now consider the solution under the ZLB. Suppose in period t agents expect that the interest rate is at the ZLB for H periods ($t = 1, \ldots, H$) and then reverts back to the normal monetary policy rule (11) afterwards ($t = H + 1, \ldots$). The log-linearized equations during the ZLB are given by

$$G^0\hat{Y}_t = G^1\hat{Y}_{t-1} + G^2E^i_t\hat{Y}_{t+1} + \Psi\hat{Z}_t, \tag{A42}$$

where the log-linearized version of the policy rule (11) is replaced with $\hat{R}_t = -\bar{R}$. For periods $t = 1, \ldots, H$, the decision rule takes a time-varying form

$$\hat{Y}_t = \varepsilon_{yy,t}\hat{Y}_{t-1} + \varepsilon_{yz,t}\hat{Z}_t$$

which implies

$$E^i_t\hat{Y}_{t+1} = \varepsilon_{yy,t+1}\hat{Y}_t + \varepsilon_{yz,t+1}(P + A^i)\hat{Z}_t \tag{A43}$$

From (A42) and (A43) we use method of undetermined coefficients to obtain

$$\varepsilon_{yy,t} = (G^0 - G^2\varepsilon_{yy,t+1})^{-1}G^1$$
$$\varepsilon_{yz,t} = (G^0 - G^2\varepsilon_{yy,t+1})^{-1}[G^2\varepsilon_{yz,t+1}(P + A^i) + \Psi]$$

starting from $\varepsilon_{yy,H+1} = \varepsilon_{yy}$ and $\varepsilon_{yz,H+1} = \varepsilon_{yz}$. Lastly we check at each period during the ZLB ($t = 1, \ldots, H$) agents are forming expectations under the worst-case beliefs.

E Accuracy of solution

I characterize the accuracy of the approximated solution using the Euler Equation Error (EEE) as in Judd (1992).[4] To understand the method, consider the capital holders' Euler equation for riskless bonds:

$$\gamma\lambda^c(s_{t-1}, \epsilon_t) = \beta d_t E^c_t\left\{\lambda^c(s_t, \epsilon_t)\frac{R(s_{t-1}, \epsilon_{t+1})}{\pi(s_t, \epsilon_{t+1})}\right\}, \tag{A44}$$

[4]See Bianchi et al. (2017) for an EEE analysis of a Markov-switching DSGE model with Knightian uncertainty. The discussion in this section largely follows theirs.

where λ^c is the capital holders' marginal utility function and R and π are the nominal interest rate and inflation, respectively. These decision rules determine the time t variables as functions of states s_{t-1} and innovations ϵ_t and the time $t+1$ variables as functions of s_t and ϵ_{t+1}. The expectations are taken under the capital holders' worst-case scenario.

Note that, under linear approximation (A44) will not hold exactly. Hence I define the Euler equation error function $EE(\cdot,\cdot)$ as

$$EE(s_{t-1},\epsilon_t) = 1 - \frac{\beta d_t \hat{R}(s_{t-1},\epsilon_t) E_t^c\left[\hat{\lambda}^c(s_t,\epsilon_{t+1})\hat{\pi}(s_t,\epsilon_{t+1})^{-1}\right]}{\gamma \hat{\lambda}^c(s_{t-1},\epsilon_t)}, \tag{A45}$$

where $\hat{R}$, $\hat{\lambda}^c$, and $\hat{\pi}$ are the computed decision rules. Judd and Guu (1997) point out that this unit-free error can be interpreted as a relative optimization error due to the agents' use of approximated decision rules. For example, $EE(s_{t-1},\epsilon_t) = 10^{-3}$ means that the agent is making a 10 cents mistake for each \$100 spent.

As emphasized in Aruoba et al. (2006), the EEE has strong theoretical foundation. In particular, Santos (2000) shows that the approximation error of the computed policy function is of the same order of magnitude as the size of the EEE, and correspondingly the approximation error of the value function is of the square order the EEE.

To compute the conditional expectation in (A45), I need to specify the parameters of the non-fiscal shock processes. For the persistence parameters, I set $\rho_z = 0.95$ and $\rho_d = 0.5$. These values are in line with the estimates found in conventional DSGE studies such as Justiniano et al. (2010). For the standard deviations, I choose the values so that the model matches the data standard deviations of real output, real consumption, and inflation. I choose $100\sigma_z = 0.2, 100\sigma_d = 0.05$, and $100\sigma_R = 0.01$.

Compared to the single-shock RBC model studied in Aruoba et al. (2006), the computation of the EEE in my model is complicated for three reasons. First, there are multiple shocks so the computation of expectations using Gaussian-Hermite quadrature points becomes infeasible due to the curse of dimensionality. Instead, the calculation of expectations requires a Monte Carlo method. Second, the fiscal instruments exhibit stochastic volatility. Third, the EEE must be evaluated under the true DGP but expectations taken under the worst-case probabilities. Specifically, I follow the steps outlined below:

1. I simulate a time series for fiscal volatility (equation (2) in the main paper). Conditional on the realization of volatility, I simulate the innovations ϵ_t which include, among other things, innovations to the fiscal instruments in equation (1) of the main paper.

2. For a given path of innovations, I use the law of motion under the true DGP to simulate a time series of the economy. This gives me the time series of the state s_{t-1}, which in turn allows me to calculate $\hat{R}(s_{t-1},\epsilon_t)$ and $\hat{\lambda}^c(s_{t-1},\epsilon_t)$ in (A45). Note that the law of motion under the true DGP already takes into account the fact that agents evaluate expectations under the worst-case scenarios.

3. To obtain the worst-case conditional expectation in (A45), I draw 2000 samples of multi-variate normal random numbers and evaluate the next period's decision rules at each point, given the current state s_t that was computed in step 2. I then take an average over the calculated decision rules to get

the conditional expectation $E_t^c\big[\hat{\lambda}^c(s_t,\epsilon_{t+1})\hat{\pi}(s_t,\epsilon_{t+1})^{-1}\big]$. Note that the decision rules are evaluated under the law of motion under the worst-case DGP (i.e., the behavior of the economy where the worst-case scenario actually realizes).

4. I now have all the ingredients needed to calculate the EEE in (A45) at each point t in the simulated time series. The current variables $\hat{R}(s_{t-1},\epsilon_t)$ and $\hat{\lambda}^c(s_{t-1},\epsilon_t)$ were obtained in step 2 and the conditional expectation in (A45) was obtained in step 3.

I follow similar steps to compute the EEEs for the capital holders' Euler equation for capital (A3) and non-capital holders' Euler equation for bonds (A23). For each equation, I simulate the economy at the posterior mode for 1100 periods (I drop the first 100 periods). To ease interpretation, I report $\log_{10}|EEE|$ instead of the raw EEE. For equations (A44), (A3), and (A23), the average absolute errors are -3.6, -3.4, and -2.1, respectively. Given that my model is larger and features many elements, such as stochastic volatility, that are absent in the baseline RBC model studied in Aruoba et al. (2006), I view these EEEs as an indication that the approximate solution used here is of reasonable accuracy.[5]

F Bayesian impulse-response-matching method

This section describes in detail the Bayesian impulse-response-matching method that is used to estimate the model. I closely follow Christiano et al. (2010b)'s description of the methodology. The first step is to compute the "likelihood" of the data from approximation based on conventional asymptotic distribution theory. Let $\hat{\psi}$ denote the impulse response function computed from the VAR and let $\psi(\theta)$ denote the impulse response function from the theoretical model, which depends on the structural parameters θ. Suppose the theoretical model as well as the VAR are correctly specified. Denote θ_0 and $\psi(\theta_0)$ the true parameter vector and impulse response function, respectively. Then we have

$$\sqrt{T}(\hat{\psi}-\psi(\theta_0)) \xrightarrow{d} N(0, Z(\theta_0)),$$

where T is the length of the sample and $Z(\theta_0)$ is the asymptotic sampling variance, which is a function of θ_0. The asymptotic distribution of $\hat{\psi}$ is rewritten as

$$\hat{\psi} \xrightarrow{d} N(\psi(\theta_0), V), \qquad V \equiv \frac{Z(\theta_0)}{T}.$$

I use a consistent estimator of V, where the main diagonal elements consist of the sample variance of $\hat{\psi}$ and the non-diagonal terms are set to zero.[6] As explained in detail in Christiano et al. (2011), this approach improves small sample efficiency and can be justified in a manner that is analogous to the estimation of frequency-zero spectral densities in Newey and West (1987). An additional advantage of this strategy is that the interpretation of the estimator is transparent and graphically intuitive: it chooses parameters so that the theoretical impulse responses lie inside a confidence interval around the empirical responses. In contrast, when the non-diagonal terms of V are non-zero, the estimator also takes into account the

[5] Bianchi et al. (2017) report EEEs that are of similar magnitudes.

[6] Christiano et al. (2005) and Altig et al. (2011) also use this approach in a frequentist context.

deviations of the model from data across different impulse responses in a way that is intractable and difficult to understand.

Next, I calculate the likelihood

$$L(\psi|\theta) = (2\pi)^{-\frac{N}{2}}|V|^{-\frac{1}{2}} \exp\{-0.5[\hat{\psi} - \psi(\theta)]'V^{-1}[\hat{\psi} - \psi(\theta)]\},$$

where N is the total number of elements in the impulse responses to be matched.[7] Intuitively, the likelihood is higher when the theoretical impulse response $\psi(\theta)$ is closer to the empirical counterpart $\hat{\psi}$, taking into account the precision of the estimated empirical responses. From the Bayes law, the posterior distribution $P(\theta|\psi)$ is

$$P(\theta|\psi) = \frac{P(\theta)L(\psi|\theta)}{P(\psi)},$$

where $P(\theta)$ is the prior and $P(\psi)$ is the marginal likelihood. I compute the posterior distribution using the random-walk Metropolis-Hastings algorithm.

G Robustness and additional experiments

I report additional robustness analysis and experiments related to the main results in Section 5.

1. Estimation using the VAR on simulated data from the model.

   Following Christiano et al. (2010a) and Christiano et al. (2010b), the target I use in the Bayesian impulse-response-matching method is the theoretical impulse response derived from the model. While this facilitates the mapping of the impulse responses from various counterfactuals to the fit of the data, it is important to check whether the finite-order VAR representation provides a good approximation of the impulse responses of the theoretical model. To this end, I estimate the model using the Sims-Cogley-Nason approach (Kehoe 2006): I estimate the parameters so that the mean impulse response from 100 structural VARs, each using 107 quarters of artificial time series generated from the baseline limited capital market participation model, fits the VAR on actual data.[8] In Figure A4, I plot the mean VAR impulse responses from the artificial time series averaged over 100 replications based on the parameters estimated using the Sims-Cogley-Nason approach. For comparison, I also plot the VAR impulse responses estimated from artificial time series generated using the counterfactual representative agent economy, where I set the share of non-capital holders χ to 0 while holding other parameters at the estimated values from the Sims-Cogley-Nason approach. Both for the heterogeneous agent and representative agent models, the impulse responses are similar regardless of whether they are estimated using the theoretical impulse responses or the Sims-Cogley-Nason approach. These findings indicate that my impulse-response-matching procedure is successful in identifying and quantifying the propagation mechanism of fiscal uncertainty shocks.

2. The role of price and wage rigidities.

[7]In my context, since I match the responses of seven variables to uncertainty shocks for four different fiscal instruments, $N = 7 \times 4 = 28$.

[8]For the parameters of the shock processes, I use the values determined in Section E.

To illustrate the role of sticky prices and wages, in Figure A5, I plot the impulse responses of the heterogeneous agent economy with flexible prices ($\xi_p = 0.1$) and the responses with flexible wages ($\xi_w = 0.1$). The impulse response in the flexible prices case (green dashed lines) is less pronounced compared to the baseline version. This underscores the importance of sticky prices: they magnify the negative income effects perceived by non-capital holders through countercyclical markups. The finding echoes conclusions from Fernández-Villaverde et al. (2015) and Basu and Bundick (2017) which emphasize countercyclical markups due to sticky prices in the transmission of uncertainty shocks. Interestingly, removing sticky wages makes the impulse response of output, consumption, and hours more pronounced (red lines with stars).

3. The role of price and wage markups.

   A different way of assessing the effect of nominal rigidities is to re-estimate the heterogeneous agent model using different calibrations of steady-state markups (Figure A6). First, when $\theta_p = \theta_w = 6$, which implies steady-state price and wage markups of 20%, the estimated response is very similar to the baseline case of $\theta_p = \theta_w = 11$. Second, when $\theta_p = \theta_w = 21$, which implies steady-state price and wage markups of 5%, consumption falls, although the fall is less pronounced relative to the baseline case of $\theta_p = \theta_w = 11$. Output and hours responses are very similar to the baseline case and investment declines more than the baseline case.

4. Effect of the zero lower bound.

   I analyze the impact of a capital income tax uncertainty shock when the economy is stuck at the zero lower bound (ZLB) on the nominal interest rate. This is a highly relevant exercise since, as noted by Baker et al. (2016) and Fernández-Villaverde et al. (2015) and others, the post-Great Recession period in the U.S. has experienced high fiscal uncertainty combined with the ZLB. To solve the economy at the zero lower bound, I extend the methodology used by Cagliarini and Kulish (2013) and Del Negro et al. (2015) to accommodate ambiguity aversion and heterogeneous worst cases. In Section D, I describe the solution procedure in detail.

   To compute the impulse responses to capital income tax uncertainty shocks under the ZLB, I follow the procedure used in Fernández-Villaverde et al. (2015). First, I hit the economy with an innovation to the household discount factor at period t_1 so that the economy is at the ZLB for $t_1 \leq t \leq t_2$. I choose the size of the innovation so that the economy is at the ZLB for five quarters. I then compare the path of endogenous variables in this economy with another economy where it experiences at period t_1 not only the discount factor shock that forces the economy to the ZLB but also a two-standard-deviations increase in capital income tax uncertainty.[9] The difference between the path of endogenous variables between the two economies thus allows me to isolate the effect of a fiscal uncertainty shock when the economy is already at the ZLB. In Figure A7, I plot such impulse responses beginning from $t = t_1$ except for the nominal interest rate, for which I report the actual realized path under the ZLB.

[9]While the algorithm allows the capital income tax uncertainty shock to alter the length of the ZLB, it turns out that the duration of the ZLB is the same with or without the uncertainty shock.

Comparing the baseline heterogeneous agent model under the Taylor rule and the baseline model under the ZLB, the output response of the latter is much larger than the response of the former. For example, under the ZLB the output drops as much as around 2.3 percent, while under the Taylor rule output drops by 0.4 percent. In my model, the effects of capital tax uncertainty are much larger because the central bank cannot lower the interest rate to counteract low aggregate demand due to the consumption cut by non-capital holders.

5. Government spending uncertainty shocks and labor income tax uncertainty shocks.

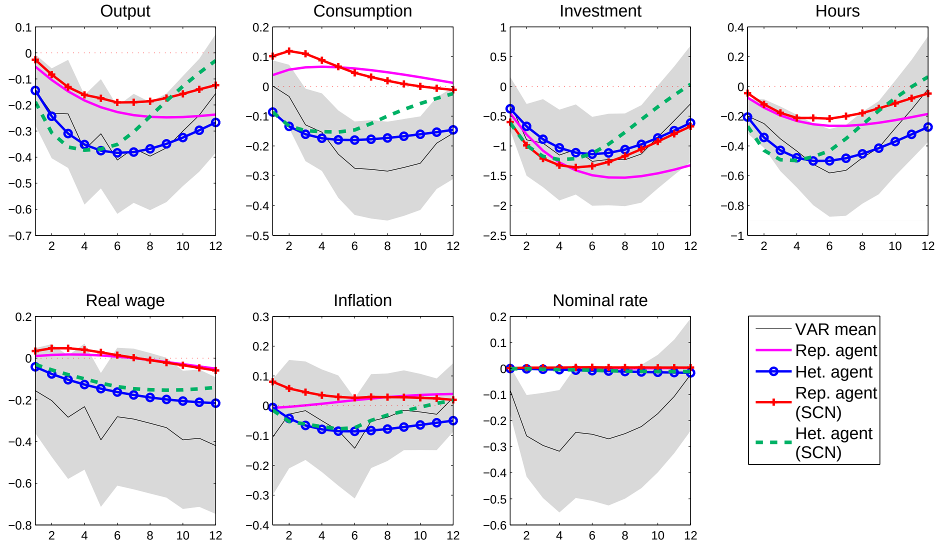
   Figure A8 reports the impulse response to a two-standard-deviations government spending uncertainty shock. In the VAR, the impulse response is insignificant for most horizons for all variables. In both representative agent and heterogeneous agent models, the shock generates less than a 0.1 percent reduction in output, consumption, and hours.

   Figure A9 reports the impulse response to a two-standard-deviations labor income tax uncertainty shock. According to the VAR, the shock reduces output by about 0.3 percent in the medium run. Consumption, investment, and hours also decline by similar amounts. Both representative agent and heterogeneous agent models quantitatively replicate the contraction in economic activity.

6. A simultaneous increase in uncertainty about all fiscal instruments.

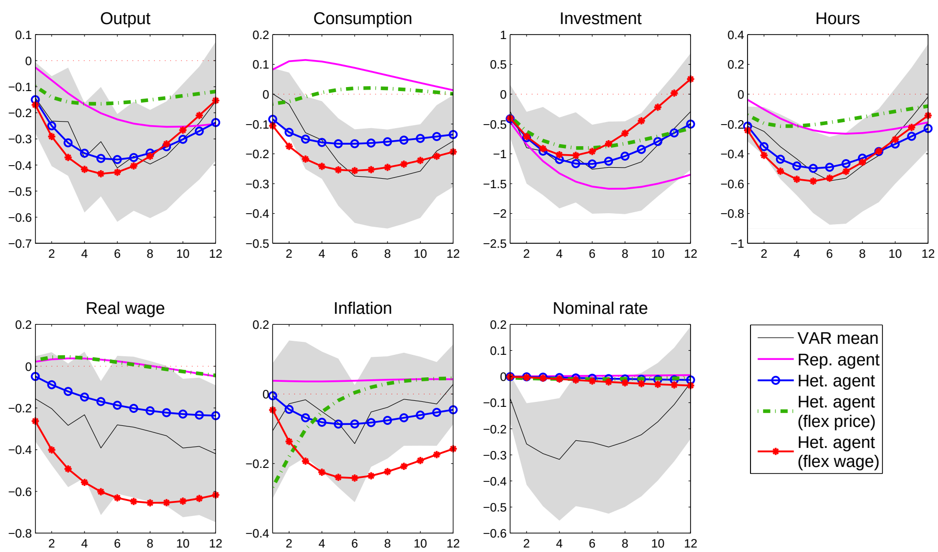
   Figure A10 reports the response of aggregate variables when there is a simultaneous two-standard-deviations increase in uncertainty about all fiscal instruments (government spending, consumption tax, labor income tax, and capital income tax). Both in the representative agent and heterogeneous agent models, an increase in fiscal uncertainty causes simultaneous decline in output, consumption, investment, and hours. Similar to the impulse responses to the capital income tax uncertainty shock, the contraction of output, consumption, and hours in the heterogeneous agent model is more pronounced than that in the representative agent model.

Figure A4: Estimated impulse response using Sims-Cogley-Nason approach



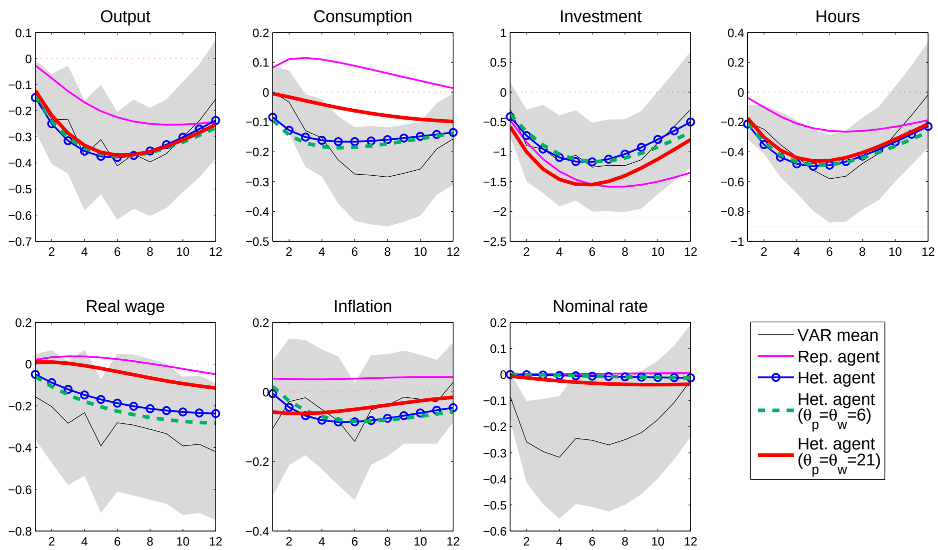
Notes: The figure reports the impulse responses to a two-standard-deviation increase in σ_{τ_k} (capital income tax volatility). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles and the purple lines are the responses from the heterogeneous agent model and the representative agent model, respectively. The green dashed lines and red lines with '+'-signs are the respective estimated impulse responses from the Sims-Cogley-Nason approach.

Figure A5: The role of price and wage rigidities



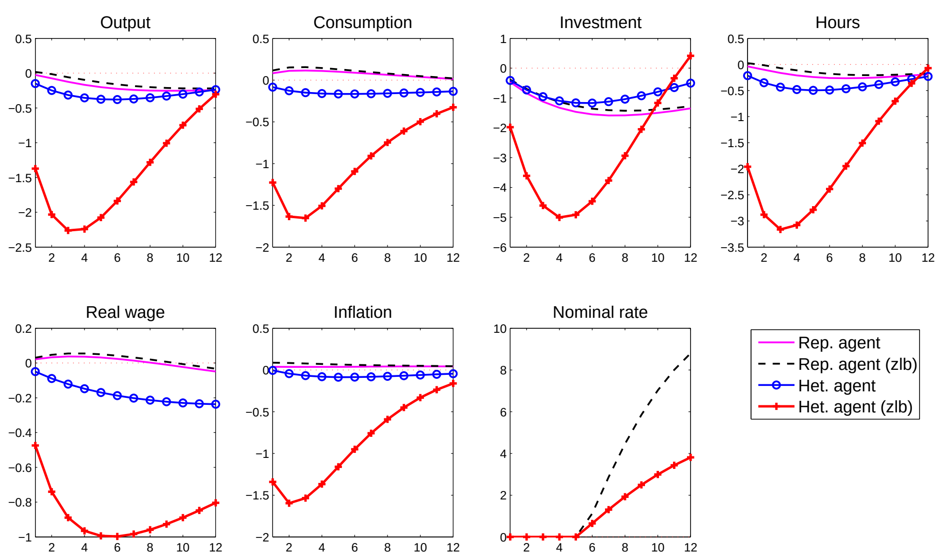
Notes: The figure reports the impulse responses to a two-standard-deviation increase in σ_{τ_k} (capital income tax volatility). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles and the purple lines are the responses from the heterogeneous agent model and the representative agent model, respectively. The green dashed and red lines are from the flexible price version and the flexible wage version of the heterogeneous agent model, respectively.

Figure A6: The role of price and wage markups



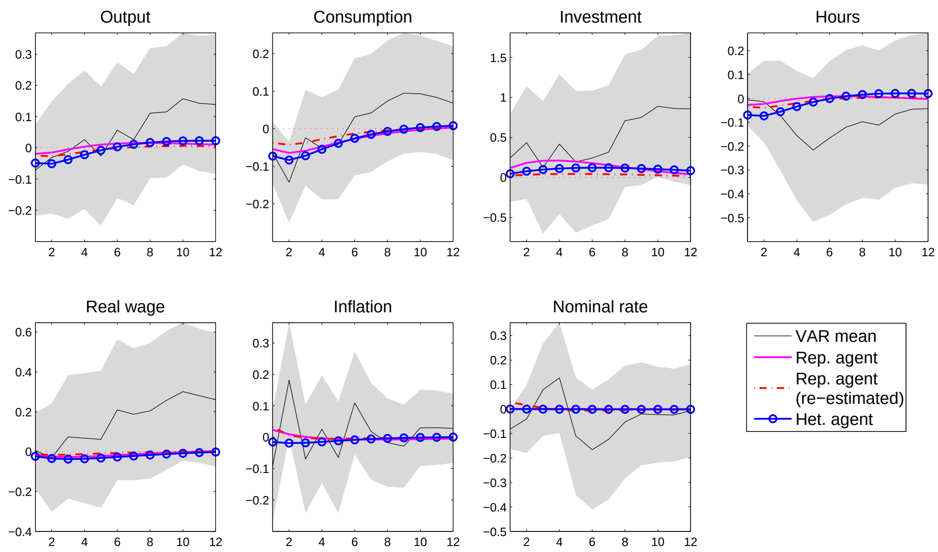
Notes: The figure reports the impulse responses to a two-standard-deviation increase in σ_{τ_k} (capital income tax volatility). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles and the purple lines are the responses from the heterogeneous agent model and the representative agent model, respectively. The green dashed lines and the red lines are the responses from the heterogeneous agent model with $\theta_p = \theta_w = 6$ and $\theta_p = \theta_w = 21$, respectively.

Figure A7: Effect of the zero lower bound



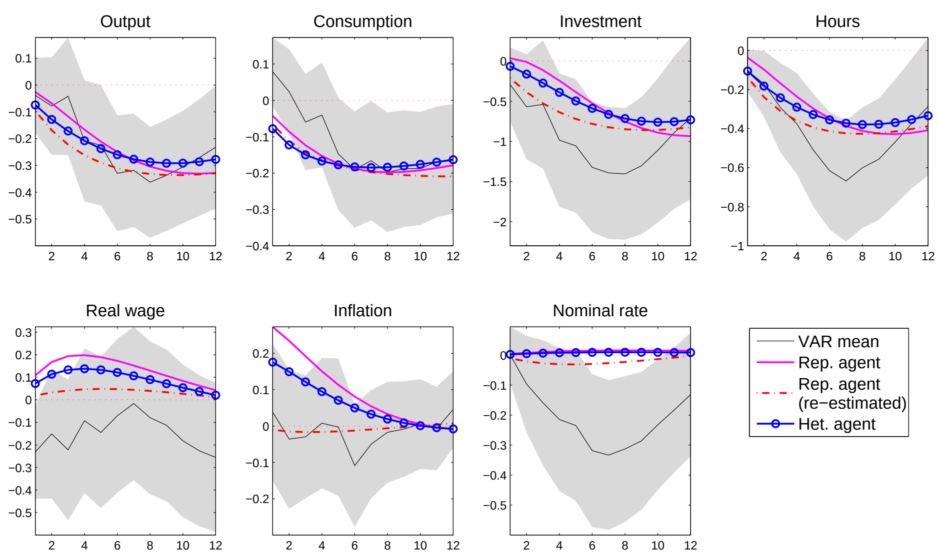
Notes: The figure reports the impulse responses to a two-standard-deviations increase in σ_{τ_k} (capital income tax volatility). The blue lines with circles and the purple lines are the responses from the heterogeneous agent model and the representative agent model, respectively. Both are under the Taylor rule. The red lines with crosses and the black dashed lines are the respective responses under the zero lower bound.

Figure A8: Government spending uncertainty shock



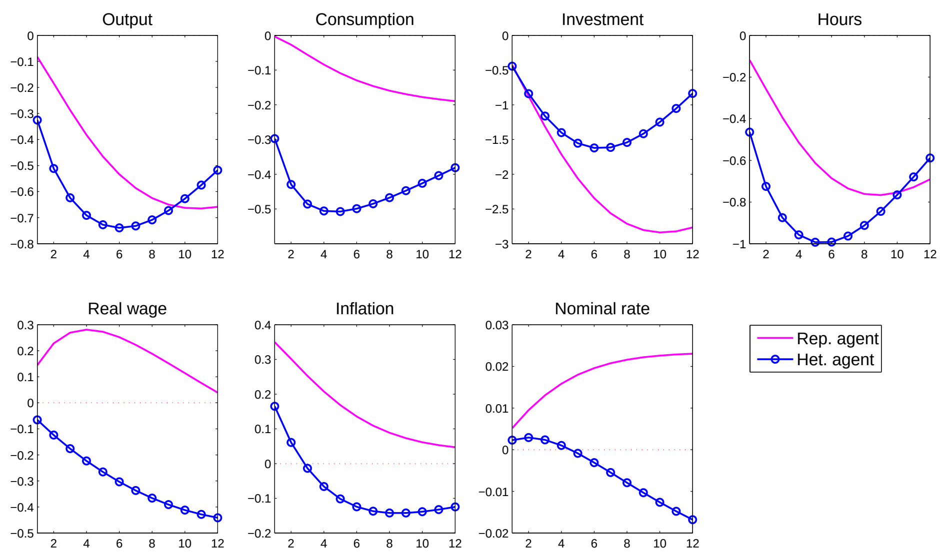
Notes: The figure reports the impulse responses to a two-standard-deviations increase in σ_g (government spending volatility). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles and the purple lines are the responses from the heterogeneous agent model and the representative agent model, respectively. The red dashed lines are the responses from the re-estimated representative agent model.

Figure A9: Labor income tax uncertainty shock



Notes: The figure reports the impulse responses to a two-standard-deviations increase in σ_{τ_h} (labor income tax volatility). The black lines are the mean responses from the VAR and the shaded areas are the 95% confidence band. The blue lines with circles and the purple lines are the responses from the heterogeneous agent model and the representative agent model, respectively. The red dashed lines are the responses from the re-estimated representative agent model.

Figure A10: A simultaneous increase in uncertainty about all fiscal instruments



Notes: The figure reports the impulse responses to two-standard-deviations increases in $\sigma_g, \sigma_{\tau_c}, \sigma_{\tau_h}$, and σ_{τ_k}. The blue lines with circles and the purple lines are the responses from the heterogeneous agent model and the representative agent model, respectively.

H Data sources

For macroeconomic variables, I use the following:

1. Real GDP in chained dollars, BEA, NIPA table 1.1.6, line 1.
2. GDP, BEA, NIPA table 1.1.5, line 1.
3. Personal consumption expenditures on nondurables, BEA, NIPA table 1.1.5, line 5.
4. Personal consumption expenditures on services, BEA, NIPA table 1.1.5, line 6.
5. Gross private domestic fixed investment (nonresidential and residential), BEA, NIPA table 1.1.5, line 8.
6. Personal consumption expenditures on durable goods, BEA, NIPA table 1.1.5, line 4.
7. Nonfarm business hours worked, BLS PRS85006033.
8. Nonfarm business hourly compensation, BLS PRS85006103.
9. Civilian noninstitutional population (16 years and over), BLS LNU00000000.
10. Effective federal funds rate, Board of Governors of the Federal Reserve System.

I then conduct the following transformations of the above data:

11. Real per capita GDP: (1)/(9)
12. GDP deflator: (2)/(1)
13. Real per capita consumption: [(3)+(4)]/[(9)$\times$(12)]
14. Real per capita investment: [(5)+(6)]/[(9)$\times$(12)]
15. Per capita hours: (7)/(9)
16. Real wages: (8)/(12)

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